Three dimensional pulsatile non-Newtonian flow in a stenotic vessel

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Abstract—This study investigates the pulsatile simulations of non-Newtonian flows in a stenotic vessel. Four non-Newtonian blood models, namely the Power Law, Casson, Carreau and the Generalized Power Law, as well as the Newtonian model of blood viscosity, are used to investigate the flow effects induced by these different blood constitutive equations. The aim of this study are three fold: firstly, to investigate the variation in wall shear stress in an artery with a stenosis at different flow rates and degrees of severity; secondly, to compare the various blood models and hence quantify the differences between the models and judge their significance and lastly, to determine whether the use of the Newtonian blood model is appropriate over a wide range of shear rates.

Keywords—fluid flows, blood, stenosis, non-Newtonian, pulsatile, simulations

I. INTRODUCTION

This paper presents the second of a two-part study on the numerical simulations of blood flow in a representative model of an arterial stenosis in the common carotid artery for various degree of severity using five blood rheological models.

The partial obstruction of arteries due to a stenosis is one of the most frequent anomalies in blood circulation. It is well known that, once such an obstruction is formed, the blood flow is significantly altered and fluid dynamic factors such as velocity, pressure or shear stress play an important role as the stenosis continues to develop [1]. So far, the specific role of these factors is not yet well understood. The ability to accurately describe the flow through a stenosed vessel would provide the possibility of diagnosing these diseases in its earlier stages. Furthermore, the presence of the anomaly itself may produce flow disturbances such as vortex formation, which has been reported as a contributing factor to atherogenesis and thrombogenesis [2].

The aim of this study are three fold: firstly, to investigate the variation in wall shear stress in an artery with a stenosis at different flow rates and degrees of severity; secondly, to compare the various blood models and hence quantify the differences between the models and judge their significance and lastly, to determine whether the use of the Newtonian blood model is appropriate over a wide range of shear rates.

2. ANALYSIS AND MODELLING

Governing equations

The blood flow is assumed to be laminar and incompressible and therefore the Navier-Stokes equations for 3D incompressible flow are given by

\[ \nabla \cdot V = 0 \] (1)

\[ \rho \left( \frac{\partial V}{\partial t} + V \cdot \nabla V \right) = -\nabla p + \tau \] (2)

where \( V \) is the 3D velocity vector, \( p \) pressure, \( \rho \) density and \( \tau \) the shear stress term.

Four different non-Newtonian blood flow models as well as the simple Newtonian model are considered in this study. The effects of these models on the flow field and the wall shear stress in the vicinity of a stenosis are examined. These models are given below [3].

Blood Models

1. Newtonian model

\[ \mu = 0.00345 \quad Pa \cdot s \] (3)

2. Power Law Model

\[ \mu = \mu_0 (\dot{\gamma})^{n-1} \] (4)

3. Casson Model

\[ \mu = \sqrt{\frac{\tau_{xy} + \sqrt{\eta \dot{\gamma}^2}}{\dot{\gamma}}} \] (5)

4. Carreau Model

\[ \mu = \mu_\infty + (\mu_0 - \mu_\infty) \left[1 + (\dot{\gamma} \lambda)^2\right]^{-\frac{n-1}{2}} \] (6)
5. Generalized Power Law Model

\[ \mu = \lambda \left| \dot{v} \right|^{n-1} \]  

(7)

where

\[ \lambda(\dot{v}) = \mu_e + \Delta \mu \exp \left[ -\left( 1 + \frac{\dot{v}}{a} \right) \exp \left( -\frac{b}{\dot{v}} \right) \right], \]

\[ n(\dot{v}) = n_e - \Delta n \exp \left[ -\left( 1 + \frac{\dot{v}}{c} \right) \exp \left( -\frac{d}{\dot{v}} \right) \right] \]

Geometry

The flow geometry comprises a tube of diameter \( D \) and can be divided into three regions, the inlet, the deformed and the outlet region. In the case of the stenosis, the lengths of these regions are \( \frac{4D}{2}, \frac{2D}{2} \), and \( \frac{D}{20} \), respectively. The radius of the undeformed inlet and outlet is \( \frac{2}{D} R \).

In the case of the stenosis, the radius of the constricted region is given by

\[ R = R_0 \left[ 1 - \left( \frac{R_0 - R_{min}}{R_0} \right) \frac{1 - \cos(\pi/D)}{2} \right] \]

(8)

where \( R_{min} \) is the minimum radius at the centre of the stenosis. In this study, three different degrees of stenosis were used, 20\%, 50\% and 80\%.

Assumptions and boundary conditions

It is assumed that the arterial walls are rigid and no-slip condition is imposed at the walls. At the outlet, stress-free conditions are applied and the pressure is set to zero. Finally, the velocity profile at the inlet is regarded to be that of fully developed flow in a straight tube and can be derived analytically for both the Newtonian and the Power Law fluids [4]. The forms are

\[ u = u_0 \left[ 1 - \left( \frac{r}{R_0} \right)^2 \right] \quad 0 \leq r \leq R_0 \]

(9)

where \( u \) is the velocity component in the \( x \)-direction for the Newtonian flow and

\[ u = u_0 \left( \frac{3n + 1}{n + 1} \right) \left[ 1 - \left( \frac{r}{R_0} \right)^{n+1} \right] \quad 0 \leq r \leq R_0 \]

(10)

for the non-Newtonian flow. In transient flow, the pulsatile flow at the inlet is given by a time varying forcing function given in Figure 1 below. This forcing function was scaled to yield a maximum inflow velocity of \( \dot{v} \) with a heart rate of approximately 60 beats per minute.

![Fig. 1. Physiological flow waveform in the carotid artery used to drive the inlet velocity boundary condition as a function of time.](image)

Solution methodology

The governing equations are highly nonlinear and must be solved numerically using techniques of computational fluid dynamics. In this study, these equations are solved using the finite element method as implemented by COMSOL (COMSOL Inc., Los Angeles, CA). The flow geometries for the stenosis was first created using Matlab. Then a finite element mesh was placed on this geometry. Briefly, an inlet plane of the artery is meshed in 2D using triangles and this mesh is extruded along the centerline of the artery to create a 3D mesh consisting of hexadrel elements. The mesh used for all computations consisted of 9,708 elements and 15,048 nodes for the stenosis.

The governing equations were solved completely using the boundary conditions for fully developed flow (9) and (10) at the inlet along with the pulsatile forcing function for the transient case.

3. RESULTS AND DISCUSSION

Transient simulations were performed using all five models given above. Three different degrees of stenosis were used namely 20\%, 50\% and 80\%.

Figure 2 below shows that all of the non-Newtonian models considered here except the Power Law model produce a higher pressure difference than the Newtonian model. Specifically, the highest pressure drop is induced by the generalized Power Law model and the lowest by the Power Law model. Similar pattern in pressure differences are obtained at higher flow rates.
The distribution of the wall shear stress (WSS) is one of the most important hemodynamic parameter due to its direct relevance in atherosclerosis formation. Figure 3 shows the distributions of maximum shear stress for various degrees of severity of the stenosis for all models. It is evident that WSS increases with increasing severity. All models show close agreement with the Newtonian model except for the Power Law model. At 50% stenosis, the WSS predicted by this model is significantly lower than the rest. Figure 4 shows the distribution of WSS along the geometry at various times. Maximum shear stresses are reached just before the throat of the stenosis. The magnitude of this value increases with higher flow rates. This peak is followed by a negative value indicating the presence of backflow. Further downstream, the WSS steadily regains its undisturbed value.

Transient simulations were performed using the Generalized Power Law Model for the stenosis and each simulation was from \( t = 0 \) to 10.0 secs, yielding a heart rate of approximately 60 beats per minute.

Figure 5 shows the distribution of maximum WSS with shear rate in a stenosis. Again, WSS increases with increasing shear rate with the Power Law model deviating significantly from the rest.
The maximum wall shear stress occurs in the middle of the cycle corresponding to the maximum inflow velocity. The distribution of shear rates in a 50% stenosed artery is shown in Figure 6. The regions of high shear are confined to the throat of the stenosis and immediately downstream of the stenosis.

The maximum and minimum WSS values are in close agreement for the Generalized Power Law, Carreau and the Newtonian models. The Power Law model gives a much lower value because it exhibits a lower viscosity at the throat of the stenosis where the shear stress is high. As the flow rate increases, these WSS differences from the first three models become less prominent indicating insignificant differences in model behavior at high shear rates.

Similar results are obtained when the diameter of the common carotid artery is assumed to be as large as 0.8 cm. The maximum wall shear stress and shear rates values are lower when compared to the 0.64 diameter artery but the differences in model behavior are analogous.

It is evident from these results that the Power Law model tends to break down at higher shear rates in that it reduces the viscosity of the blood to levels below the Newtonian level which theoretically should not be possible. This is noticeable in the 80% stenosed model. The pressure difference predicted by this model is less than the Newtonian model, indicating a lower than Newtonian viscosity. This method also shows very low wall shear stress levels, dropping below Newtonian levels at fairly low shear rates, for example at the medium flow rate at 50% stenosed, the WSS levels are less than Newtonian levels. This model is relatively easy to use but predict decreasing viscosity at higher strain, contrary to the generally accepted observation that blood behaves as a Newtonian fluid for strains above \(200 \text{ s}^{-1}\). At low shear rates the Casson model shows near Newtonian behavior with the behavior becoming less Newtonian as the shear rate increases for a period, then reverting to near Newtonian behavior as the rate continues to increase. This model takes the haematocrit factor \(H\) (the volume fraction of red blood cells in whole blood) into account, with the parameters given (obtained from data fitting) suggesting a value of \(H\) of 37%. However, it is reported that this yields a limiting viscosity at high shear slightly above the usual Newtonian value. The results obtained here suggest the same with WSS values above the Newtonian values at low and very high shear rates. This model appears to be fairly accurate at high shear rates, but not at low shear rates.

The Carreau model produces values that are in close agreement with that of the Newtonian model at shear rates well above \(200 \text{ s}^{-1}\). Our results indicate this to be the case. Both the WSS and the pressure difference tends to the Newtonian values at shear rates in excess of \(3000 \text{ s}^{-1}\). This model by design reverts to Newtonian numbers as shear rates approach infinity. The basis for this model is the constant Newtonian viscosity, modified to non-Newtonian such that the modification tends to zero as the limit of the shear rate goes to infinity.

Finally, the Generalized Power Law model gave results that are in closest agreement with the Newtonian values at mid-range and high shear rates. At low shear rates, this model gives values that are close to that of the Power Law and the Carreau models. While the Power Law model breaks down at high shear, our results show a close agreement between the Generalized Power Law and the Carreau models even at high shear rates as shown in Figure 5. The Generalized Power Law model is widely accepted as a general model for non-Newtonian blood viscosity. It includes the Power Law model at low shear rate and the Newtonian model at mid-range and high shear rates. There is also good agreement between the Generalized Power Law and the Carreau model for low shear rates.

4. Conclusions

A study of the effects of modeling blood flow through a stenosis using five different blood rheological models is presented. The flow field and wall shear stress distributions produced by each model are investigated for various flow rates and degrees of abnormality. The results show that there are significant differences between simulating blood as a Newtonian or non-Newtonian fluid. It is found that the Newtonian model is a good approximation in regions of mid-range to high shear but the Generalized Power Law model provides a better approximation of wall shear stress at low shear.

162
These conclusions are presented under the assumption that the arterial walls are rigid and zero pressure is assumed at the outlet. A more realistic simulation would include elastic walls and incorporate the effects of upstream and downstream parts of the circulatory system into the boundary conditions. This is a long term objective of this study.

References:


