Relative Risk Aversion with Loss Aversion

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Abstract

I introduce loss aversion with an endogenous reference point in the context of state-dependent recursive preferences. With loss aversion, the representative agent’s relative risk aversion coefficients are lower. But the stochastic discount factor calibration does not match its implied values.

Keywords: Loss aversion, Relative risk aversion, State dependent recursive preferences, Equity premium

“A penny saved is two pence dear” ~ Benjamin Franklin

I. Introduction

Rajnish Mehra and Edward Prescott (1985) introduced the equity premium puzzle in the context of a two-state Markov model for real per capita consumption growth with constant relative risk aversion utility function. They found in competitive pure exchange economies the average real annual yield on equity should be a maximum of four-tenths of a percent higher than that on short term debt, in sharp contrast to the six percent premium historically observed.

Since then, many economists addressed this puzzle in various ways. Behavioral models were developed to explain the violations of expected utility paradigm in the context of asset pricing. Daniel Kahneman and Amos Tversky in 1979 introduced the prospect theory. They defined and found empirical support for a model of preferences with loss aversion. Loss aversion refers to the cases in which the agent is more sensitive to losses than to gains. This creates asymmetry in his behavior. This differentiation does not exist in standard asset pricing models. Previous research papers analyzed the impact of preferences with loss aversion on asset prices; see Shlomo Benartzi and Richard Thaler (1995), Nicholas Barberis et al. (2001), Barberis and Ming Huang (2009) among others.

The idea that prior outcomes may affect subsequent risk taking behavior is supported by another strand of the psychology literature. For example, Thaler and Eric Johnson (1990) found, when faced with sequential gambles, agents are more willing to take risk if they made money on prior gambles than if they lost. So, losses are less painful when occur after prior gains and more painful when follow prior losses.

In fact, state dependent preferences addresses the same idea in a different way. It allows the parameters to vary across states of nature. Hence, it differentiate the value of the parameters depending on each state. State dependent recursive preferences were studied in Angelo Melino and Alan Yang (2003) and Sara Nada (2013b).

On the other hand, combining behavioral models in general with recursive utility gave rise to interesting results. Some authors adopted this approach before; for example, Routledge and Zin (2010), but they studied the generalized disappointment aversion instead of loss aversion as a behavioral model. In addition, Andries (2012) studied loss aversion with recursive preferences. She introduced them in a different context other than state dependent preferences though.

Given the financial and economic crises which happened through the time period 1889-2009, hereafter, I would like to explore the research questions: What does loss aversion imply for the risk aversion of the representative agent? Does taking into consideration loss aversion help in lowering his value of risk aversion?

In this research paper I would like to work both on the preferences and the behavioral based models in approaching the equity premium puzzle. It is at the same time close to the approach of disastrous events, though dealing with the idea from a different perspective.

The main findings can be summarized in few facts:
Loss aversion results depend on the representative agent’s degree of loss aversion. Elasticity of intertemporal substitution does not vary much across states with the introduction of loss aversion in the model, the values almost coincide. In addition the coefficient of relative risk aversion in the high state of nature is approximately equal to one for all degree’s of loss aversion.

Hansen- Jagannathan bounds are satisfied on average, but not for the high current state of nature.

Although loss aversion helps in lowering the relative risk aversion coefficient, in my modest opinion, state dependent preferences without the introduction of loss aversion could match better the stochastic discount factor values with a mixed risk aversion agent.

II. Theoretical Framework

A. The Environment

Using the environment setting as in Nada (2013b), preferences are temporal Von Neumann Morgenstern state dependent recursive preferences; just I will introduce loss aversion in the context. I consider an infinitely-lived representative agent, who receives utility from the consumption of a single good in each period. The economy is frictionless. At time $t$ current consumption is non-stochastic but future consumption levels are generally stochastic. There are two assets, one risky and another risk free. The equity share is completely traded. There are two states of nature; a high and a low one. The low state of nature represents cases like economic and/or financial crisis. Instead, a high state can refer to normal economic conditions or expansions. I assume both states of nature are equally likely to happen with a symmetric transition matrix as follows:

$$
\begin{pmatrix}
\pi_{ll} & \pi_{lh} \\
\pi_{hl} & \pi_{hh}
\end{pmatrix}
$$

Where $l$ and $h$ refer to the low and the high state respectively. $\pi_{ij}$ is the probability of going from state $i$ to state $j$.

I shall be as close as possible to Mehra-Prescott environment. It usually refers to the environment introduced in Mehra and Prescott (1985); a Lucas endowment economy in which real per capita consumption growth is a Markov stationary process. Under the basic consumption based model, the only state variable is real per capita consumption growth rate.

I assume the subjective discount factor $\beta$ is constant. On the other hand, other variables are state dependent. Other variables include; risk aversion, elasticity of intertemporal substitution, real per capita consumption growth, the risk free rate and price-dividend ratio.

B. Preferences:

A loss averse agent values consumption outcome relative to a reference point. In this case, the agent suffers additional disutility if the realization next period disappoints. Hence, The certainty equivalence will be a pair wise function; depending on whether the agent’s value function is above or below a reference point. The reference point is the point below which the agent suffers disutility. As in Andries (2012), the reference point is endogenous. With loss version, the agent will have a kinked utility function.

As mentioned in section A, there are only two states of nature. Then, it is logical to think the representative agent is loss averse in the current low state of nature while he behaves normally in the high one. In other words, the utility function is expected to be lower than the reference point when the current state of nature is the low one.

The representative agent has recursive preferences as in equation (2.1).  

$$
U_t = \left( c_t^{\rho(s_t)} + (\beta(\mu_{t}^{LA})^{\rho(s_t)})^{\frac{1}{\rho(s_t)}} \right)
$$

With a certainty equivalence function $\mu_{t}^{LA}$ as follows:

$$
\mu_{t}^{LA} = \begin{cases} 
(E_t(U_{t+1}^{u_{t+1}}))^{\frac{1}{\alpha_l}} & U_{t+1} < Ref_t \\
(E_t(U_{t+1}^{\alpha_l} * Ref_t^{\alpha_l-a}))^{\frac{1}{\alpha_l}} & U_{t+1} \geq Ref_t
\end{cases}
$$

where $E_t$ is the expectation conditional on time $t$ information, $c_t$ refers to the representative agent’s real consumption at time $t$, $\beta$ represents the agent’s subjective discount factor, $\alpha$ indicates the level of risk aversion of the representative agent without loss aversion , $\alpha_l$ represents the coefficient of risk aversion in the case when loss aversion condition applies; i.e. when $U_{t+1}$ is lower than the reference level while $\rho(s_t)$ indicates the time preference of the agent. More precisely, the coefficient of relative risk aversion for ‘timeless gambles’ is $1-\alpha$ and $1-\alpha_l$, without and with loss aversion respectively; see Larry Epstein and Stanley Zin (1989). While the elasticity of intertemporal substitution is $1/(1-\rho)$. $\alpha$, $\alpha_l$ and $\rho(s_t)$ depend on an exogenous state variable $s_t$.

Elaborating more, $\alpha_l$ is expected in general to be higher than the case when there is no loss aversion. Having different parameters, with and without loss aversion, leads to having different slopes of the certainty equivalent function in each case. The degree of the
agent’s loss aversion affects the ratio of these two slopes in the following way:

\[ \frac{a}{a_l} = 1 - \gamma \]  

(2.2)

Where \( \gamma \) represents the coefficient of loss aversion.

The reference point is defined, as in Andries (2012), to be:

\[ Ref_t = e^{E_t[\log(U_{t+1})]} \]

C. Budget constraint:

Following Melino and Yang (2003) theoretical model, \( x_t \) represents beginning-of-period wealth,

\[ x_t = (d_t + p_t)x_t \]  

(2.3)

And wealth evolves according to the following formula

\[ x_{t+1} = (x_t - c_t)w_t r_{t+1} \]  

(2.4)

Where \( w_t \) is the portfolio share and \( r_{t+1} \) is the gross return.

The representative agent will be constrained by his own level of wealth as in equation (2.4). Hence, he will maximize his recursive utility preferences, subject to the budget constraint.

D. Equilibrium with Loss Aversion:

Maximizing \( U_t \) subject to the wealth accumulation constraint given in equation (2.4) yields the Euler equations as first order conditions:

\[ E_t(Q_{t+1}M_{t+1}) = 1 \]  

(2.5)

\[ E_t(Q_{t+1}r_{t+1}) = 1 \]  

(2.6)

Where \( M_{t+1} = w_t^r r_{t+1} = \sum w_t^r r_{t+1} \) is the gross return to holding the optimal portfolio \( w_t^r \) from time \( t \) to \( t+1 \). \( Q_{t+1} \) refers to the stochastic discount factor.

Introducing loss aversion in the representative agent problem affects the formula of the stochastic discount factor. Following the same settings of Melino and Yang (2003) and under the same equilibrium conditions, the Stochastic Discount Factor will be:

\[
Q_{t+1} = \begin{cases} 
\beta g_{t+1} \frac{E_{t+1}}{E_{t}} \left( 1 + PD_{t+1} \right)^{-\alpha} & \text{for the current low state} \\
\beta^{\gamma_{t+1}} g_{t+1} \left( PD_{t+1} \right)^{-\frac{1}{1-\gamma_{t+1}}} \left( 1 + PD_{t+1} \right)^{-\frac{n}{n-1}} & \text{for the current high state}
\end{cases}
\]

(2.7)

While the reference point at equilibrium will be:

\[ Ref_t = e^{E_t[\log(\frac{\beta g_{t+1} \left( 1 + PD_{t+1} \right)^{-\alpha}}{PD_{t}})]} \]  

(2.8)

III. Motivation, Methodology and Data

A. Motivation

The significance of state dependent preferences has been stressed in Melino and Yang (2003) and Nada (2013b) among others. Its main importance resides in allowing the stochastic discount factor to vary across states, being sensitive to the current state as well as the next period’s state.

I would like to concentrate more here on loss aversion. Some simple questions might arise as: what is the motivation for adding loss aversion? Will it add to the state dependent preferences assumption? What difference loss aversion can make?

Although state dependent preferences were insightful about the stochastic discount factor, the average value for the relative risk aversion is still outside the acceptable levels considered throughout the literature; see Nada (2013b).

Introducing loss aversion affects the stochastic discount factor formulation. So it will affect the risk aversion parameter in a different way. An interesting question might be: can introducing loss aversion help in lowering the relative risk aversion coefficients given the different formulation for the stochastic discount factor?

3 please refer to Andries (2012) for more details.

4 Please refer to Nada (2013a).
With the introduction of loss aversion in the settings, the difference between both coefficients for the relative risk aversion in different states will be due to the loss aversion effect. When loss aversion condition is satisfied, the agent’s risk aversion is supposed to be higher than in normal cases. Taking into account loss aversion coefficient might result in lower values for the coefficients of relative risk aversion. Hoping these values result within acceptable limits in the literature. Meanwhile, no study has examined the case of loss aversion neither in the context of Mehra-Prescott settings nor with state dependent preferences up to the author’s knowledge.

Thus I was motivated by the aforementioned points together to study state dependent recursive preferences’ and loss aversion’s effects on explaining equity premium facts. And if loss aversion can add to this explanation in the context of state dependent recursive preferences.

B. Methodology

I will be using the same methodology for state dependent preferences in Nada(2013b), but applying it on the loss aversion case. The subjective discount rate is fixed. Both the risk aversion and elasticity of intertemporal substitution vary across states. The variation of the coefficient of relative risk aversion across states will be due to loss aversion. When the current state of nature is the low one, the agent will be loss averse, so his relative risk aversion coefficient will be higher than the normal case. I will look for the values of both parameters that will solve the system of Euler equations; equations (2.4) and (2.5). Hence, I didn’t impose any restrictions neither on the coefficient of risk aversion nor on the one of elasticity of intertemporal substitution. In contrast, there is an additional parameter in the context of loss aversion; which is the loss aversion coefficient $\gamma$. I will try different values for $\gamma$ and check its effect on the estimated parameters.

I would like to examine the results of two datasets as in Nada (2013b) for the results to be comparable. First, I would like to re-examine the same sample 1889-1978, then secondly the whole sample from 1889 until 2009. The reason why the data finishes at 2009 is that the latest consumption data available was at this date.

In all my calculations I was using Matlab.

C. Data

The dataset is on the United States Stock Market index; more precisely the Standard and Poor’s composite price index “S&P 500”. The whole sample is of 121 years from 1889 till 2009 with annual frequency. While the subsample starting from 1889 until 1978 includes 90 years. The data before 1969 is from the website of Prof. Mehra. While for the rest of the sample; starting from 1969 until 2009, prices of S&P 500 and the three-month Treasury Bill rates are from Data Stream. Consumer price index data is from the World Bank with a base year 2005. On the other hand, data in the consumption per capita on non durables and services is from the Bureau of Economic Analysis “BEA”.

Variables used in the paper are as follows:

(a) A real risk free rate: it is a real yield on a relatively riskless short term security. Prior to 1920, the proxy for the risk free security was two-months to three-months Prime Commercial Paper. While form 1920-1930 Treasury Certificates represented the riskless security. For the rest of the sample the three-month Treasury Bill rate was used instead.

(b) Real return of S&P 500 index: it serves a proxy for the return of a risky asset representing the market. Returns are calculated from the annual Standard and Poor’s Composite stock price index “S&P 500” adjusted for dividends.

(c) Real consumption per capita growth: the growth rate of a real per capita consumption on non durables and services series.

(d) Real equity premium: is the annual excess real return of the S&P 500 over the real risk free rate.

(e) Consumer Price index: these series are used to calculate the consumer price index growth rate as a proxy for inflation rate. Then using these rates to get the real values of the mentioned economic series. The Consumer Price Index used is with base year 2005.

Throughout the research paper, the variables are in real values.

IV. Empirical Results

A. State Dependent Preferences with Loss Aversion (1889-1978)

The time period 1889-1978 is the one originally examined in the seminal paper of Mehra and Prescott (1985) then in Melino and Yang (2003) with state dependent recursive preferences and revisited in Nada(2013b). In this section I would like to study first this subsample. With loss aversion there is an extra parameter which is the loss aversion coefficient $\gamma$. Kahneman and Tversky (1979) estimated the ratio of
the slopes; i.e. the loss aversion coefficient, to be 0.44\(^5\) (1/2.25). But I would like to demonstrate the results for a range of values for \(\gamma\), including the value estimated by Kahneman and Tversky (1979). This range includes values of \(\gamma\) equal to 0.05, 0.15, 0.25, 0.36, 0.44, 0.5, 0.56 and 0.75.

Given \(\gamma\) equivalent to 0.44 and 0.5 were best satisfying the Euler equations in this subsample, I will report their results in details; see table 1.

**Table 1. Results for the subsample (1889-1978)**

<table>
<thead>
<tr>
<th>(\gamma)</th>
<th>Low state</th>
<th>High state</th>
<th>Low state</th>
<th>High state</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.44</td>
<td>1.34</td>
<td>1</td>
<td>18.2</td>
<td>0.99</td>
</tr>
<tr>
<td>0.5</td>
<td>0.99</td>
<td>0.99</td>
<td>0.6</td>
<td>0.57</td>
</tr>
</tbody>
</table>

In the normal case, i.e. when current state is the high one, the representative agent is modestly risk averse, with a relative risk aversion approximately equal to 1 in both cases as reported in table 1. When the agent is loss averse, the relative risk aversion coefficient depends on the coefficient of loss aversion value. With \(\gamma = 0.44\), the agent will have a slightly higher risk aversion coefficient than the current high state case. On the other hand, when \(\gamma = 0.5\), the relative risk aversion coefficient will go up to be 18, and there will be a significant difference in the risk aversion across states. The overall average relative risk aversion parameter with \(\gamma\) equals to 0.44 and 0.5 is 1.2 and 9.6 respectively. Both are within the acceptable levels. With state dependent recursive preferences, the average relative risk aversion coefficient was 18 in the same time period.

The elasticity of intertemporal substitution almost doesn’t vary across states. It has a slightly lower value for \(\gamma = 0.5\). For these two values of loss aversion coefficients, the elasticity of intertemporal substitution values are closer to 1 than to zero.

Let’s now have a look at the model’s stochastic discount factor values and comparing them with the implied stochastic discount factor from data; presented in table 2.

As in the case of state dependent preferences, the introduction of loss aversion could not either capture the negative value for the implied stochastic discount factor going from a low to a high state. When the agent is loss averse the values are close to the implied ones, though not equal. While for a current high state of nature, the model’s values for the discount factor does not vary significantly across states. Besides being countercyclical which is not the case implied values from the data.

What about the other values for \(\gamma\)? How will results differ? As can be seen from Figure (1) below, with the pair wise formulation for the stochastic discount factor, the equilibrium values of \(\alpha\) across different values for the loss aversion coefficient are very close; they vary between 0.7 and 1. These differences are not noticeable in figure (1).

In contrast to this, the value of \(\alpha_t\) depends on the loss aversion coefficient being used. For low values of \(\gamma\), \(\alpha_t\) will have modest values while for values of \(\gamma\) greater than 0.44, \(\alpha_t\) will indicate high levels for the relative risk aversion coefficient.

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\(^5\) In Benartzi and Thaler (1995) with myopic loss aversion \(\gamma\) was estimated to be 0.36.

\(^6\) Nada (2013b).
The agent being loss averse, elasticity of intertemporal substitution are almost the same across both states of nature, as is demonstrated in figure (2). On the other hand, whether the elasticity of intertemporal substitution values are close to zero or far from it depends clearly on the level of loss aversion.

B. State dependent preferences with Loss Aversion
(1889-2009)

In the whole sample set (1889-2009), values of \( \gamma \) best satisfying the Euler equations are 0.5 and 0.75 instead. Parameter results for these two values are shown in table 3.

Table 3. Results for the subsample (1889-2009)

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Low state</th>
<th>High state</th>
<th>Low state</th>
<th>High state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Risk Aversion</td>
<td>36.12</td>
<td>0.98</td>
<td>47.87</td>
<td>0.99</td>
</tr>
<tr>
<td>Elasticity of Intertemporal substitution</td>
<td>0.25</td>
<td>0.14</td>
<td>0.1</td>
<td>0.03</td>
</tr>
</tbody>
</table>

In this time period 1889-2009, with \( \gamma = 0.5 \), the high state representative agent’s relative risk aversion coefficient is approximately the same as in the subsample. While it is much higher for the low state of nature. The high relative risk aversion coefficient does not vary much with different values of \( \gamma \). The overall average relative risk aversion coefficient is 18.6 and 24.4 for \( \gamma \) equals 0.5 and 0.75 respectively. This to be compared with the average relative risk aversion with state dependent preferences which was 20.5 in the same time period.

Unlike the subsample results, the elasticity of intertemporal substitution value is close to zero and varies across states.

On the other hand, the stochastic discount factor values for the low current state are even closer to the implied values. While its values for the current high state of nature are the same as in the subsample yet.

From Figure (3), the same relation is detected, as previously in the subsample, between loss aversion and relative risk aversion coefficients. The difference between the representative agent’s risk attitude when current state is the low or the high one depends on the degree of his loss aversion.

![Figure(2): Elasticity of Intertemporal Substitution (1889-1978)](image)

![Figure(3): Risk aversion values in both low and high states (1889-2009)](image)
Still the values for the elasticity of intertemporal substitution almost coincide across states with the exception of loss aversion coefficient of values 0.5 and 0.75; please refer to figure (4).

V. Conclusion

Introducing loss aversion in the context of state dependent recursive preferences relates the difference between state dependent relative risk aversion coefficient to the fact that agent is loss averse in the low state. In this context as well, the stochastic discount factor depends on both current and upcoming state of nature. It has the highest value when both states of nature; the current and the next one, are low.

On the other hand, the stochastic discount factor is satisfying Hansen-Jagannathan bound in both samples on average. But looking at the case when the current state is the high one, the bound will not be satisfied. This is clear from its values as they do not vary much, while data implies a significant variation for the stochastic discount factor values across states. In other words, the model’s stochastic discount factor values given a high current state of nature, approximately does not depend on the state of nature that will realize in the next period.

Meanwhile, the elasticity of intertemporal substitution coefficient is almost constant across states. Whether its value is low or high; i.e. close to zero or one, depends on the loss aversion coefficient being used.

Turning to the relative risk aversion coefficient, how the overall average of the representative agent’s risk aversion vary across different levels of an agent’s loss aversion is plotted in figure (5).

As it appears from the graph, the overall average of relative risk aversion lies within the acceptable limits when the loss aversion coefficient is below 0.5 for the subsample and 0.44 for the whole sample.

Loss aversion might lead to lower values for the relative risk aversion coefficient, but it could not match the implied stochastic discount factor as with state dependent preferences; see Nada(2013b).

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