Piezoelectric based vibration energy harvester with tip attraction magnetic force: modeling and simulation

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***Abstract*— In recent times, vibration based energy harvesting has drawn attention of many researchers worldwide. This is due to advancement in Microelectromechanical (MEM) devices and wireless sensors with power requirements in range of microwatts-milliWatts; hence vibration energy sources have promising potentials for such demands. Thus, many attempts were made by researchers to develop different system configurations for effective utilization of vibrations that happen all around us. Cantilever beams with piezoelectric patches are used to harness kinetic energy from mechanical vibrations. This paper is aimed at studying the effect of adding a magnetic force at the tip mass, which is also a magnet with opposite pole, on power output of the energy harvester. The additional attraction force between the tip mass and the fixed magnet influences stiffness of the system, whilst tuning the natural frequency. Mathematical equation to depict the relationship between tuned natural frequency and distance between permanent magnet is derived. The lumped parameter model for the harvester with single fixed magnets aligned with magnetic tip mass is derived. MATLAB software is used to perform the simulation study on influence of the magnet on the harvester.**

Keywords*— *Piezoelectric; Magnet; Vibration energy harvester

I. Introduction

Recent interest in developing micro and macro electromechanical systems for harvesting ambient vibration energy has increased immensely as reviewed in [1, 2]. This is as a result of rapid development in technology of small electronic devices with very low power requirements. Piezoelectric based vibration energy harvester among other forms of transduction mechanisms such as electromagnetic and electrostatic harvesters [3] has stood the test of time in providing required energy to power these devices. Due to that, many efforts were made by researchers to develop mathematical models of different piezoelectric, beam configurations for effective estimation of power output of the harvesters. Diyana et al.[4] derived the mathematical model of a single and comb-shaped piezoelectric beam structure based on Euler-Bernoulli beam theory. The model used to estimate the influence of the proposed structure (i.e. Comb-shaped beam) in generating voltage output when excited at the base. Erturk et al.[5] proposed a model of an L-shaped piezoelectric beam energy harvester. This configuration is modeled as a linear distributed parameter model for predicting the electromechanical couple voltage response and displacement response for the energy harvesting structure. De Marqui junior C.et al.[6] presented an electromechanical coupled finite element (FE) plate model for the estimation of power output of piezoelectric energy harvester plates.

It is noteworthy that Modeling techniques are not limited to the aforementioned; several other approaches are available for the derivation of mathematical models of piezoelectric beam energy harvester configuration. Rayleigh Ritz method, Hamilton Principle and single degree of freedom (SDOF) to mention but few, are some of the techniques that can be employed to derive the equation of motion of harvesters for good estimate of their output power. In view of that, SDOF model is adopted in this paper for developing the model of the proposed harvester. Although, some literatures adopt continues system model approach (for example [5, 7]) to develop the equation of motion of the piezoelectric beam structures, however, higher vibration mode yield low magnitude of the voltage output and are usually far away from the fundamental mode.[3, 8]. Therefore, SDOF model is sufficient enough for estimating the power output of a piezoelectric energy harvester (PEH).
Magnetic interactions are extensively used in piezoelectric energy harvesters (PEHs) to achieve a broadband frequency bandwidth as reported in [9-13]. The permanent magnets are often used to tune the natural frequency of the harvesting system so as to coincide with the excitation frequency (as it is a well known fact that maximum output of PEH is achieved at resonance frequency). Challa et al.[9] employed magnetic force to tune the resonance frequency of a harvester by altering its stiffness. The harvester consists of a spring-screw as tuning mechanism which was used to alter the distance between aligned magnetic forces. Ayala-Garcia

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et al.[13] proposed a vibration energy harvester(VEH) by employing same tuning mechanism used by[9]. The novel kinetic energy harvester uses close loop control to detect the phase difference between the base and the harvester which in turn, detects the direction to tune the magnet. Also, the non- linearity introduced by magnet interactive force can benefit wideband frequency as well (for example [10-12]). In spite of the fact that, wide frequency bandwidth have been achieved in the reported literatures [9-13] by employing magnetic interactions in the proposed PEHs. However, the effect of magnetic force on the power out has not been addressed.

The main focus of this paper is to study the effect of magnetic force on the power output of a PEH subjected to base excitation. A model of a conventional PEH and a single fixed magnets aligned with magnetic tip mass of PEH are used for the study. For efficient utilization of magnetic force in widening the bandwidth of PEHs, through resonance tuning approach, an equation relating the distance for a given tuned resonance frequency is also derived.

II. Mathematical Modelling Of The Piezoelectric Energy Harvesters

A. Conventional Cantilever PEH

A typical configuartion of a conventional PEH is shown in Fig .1, with a piezoelectric layer bonded to the root of an aluminum beam with tip mass (M_t) subjected to base excitation $x_0(t)$. The sytem can be modelled as a SDOF system depicted in Fig. 2. The discrete parameter electromechanical equation of PEH in Fig. 2 is derived by applying Newton's second law in the mechanical domain and Kirchoff's law in the electrical domain[5]

$$M_{eq}\ddot{x}(t)+C_{eq}\dot{x}(t)+K_{eq}x(t)+\Theta_b V(t)=-\mu M_{eq}\ddot{x}_0(t) \tag{1}$$

$$I(t)+C^S\dot{V}(t)-\Theta_f\,\dot{x}(t)=0 \tag{2}$$

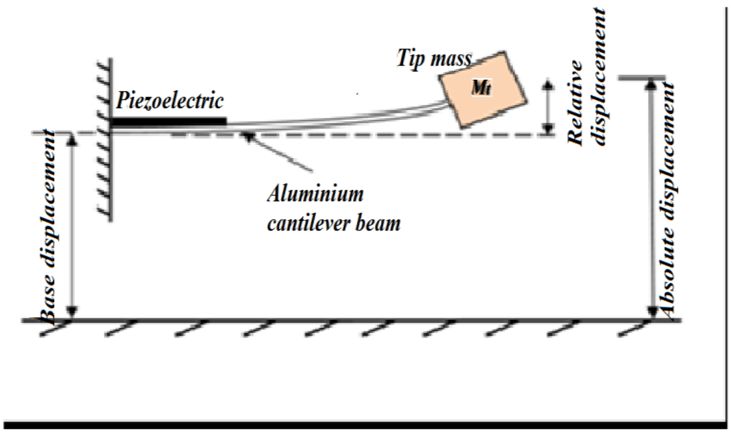

Figure 1 A conventional piezoelectric Harvester (PEH)

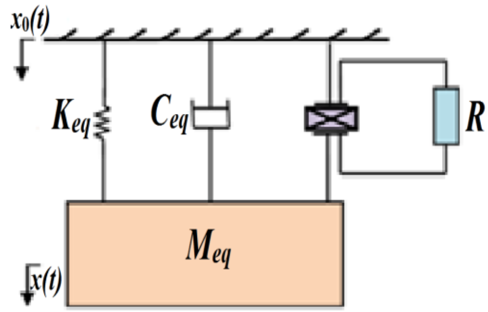

Figure 2 A single degree of freedom model of PEH

Equation (1) and (2) represent mechanical and electrical domain of the PEH respectively, where $x(t)$ is the relative displacement of the tip mass (M_t) and $M_{eq}, C_{eq},$ and K_{eq} are equivalent mass, damping and stiffness of the PEH, respectively; $\Theta_b V(t)$ is the force feedback of the induce voltage while (Θ_b and Θ_f are forward and backward electromechanical effects respectively). μ is amplitude correction factor for improving the lumped parameter model [14]

$$\mu=\frac{\left(M_t/M_b\right)^2+0.603\left(M_t/M_b\right)+0.08955}{\left(M_t/M_b\right)^2+0.4637\left(M_t/M_b\right)+0.0571} \tag{3}$$

Where $\left(M_t/M_b\right)$ is the ratio of the tip mass to distributed mass of the beam; $I(t)$ and $V(t)$ are the current and the voltage output from the PEH; C^S is the clamp capacitance of the piezoelectric transducer. From ohms law, $V(t)=I(t)R$, therefore $I(t)$ in (1) can be substituted by $V(t)/R$; where R can be considered as the load resistance.

B. PEH with magnetic tip mass aligned to fixed magnet at enclosure

To develop the model of the PEH configuration shown in Fig. 3 , having a fixed magnet aligned with magnetic tip mass in an enclosure (where the Magnets are used to implement resonance tuning by adjusting the distance D_o or introduce non linearity in the harvester for broad bandwidth frequency [9-13]), the electromechanical coupling equations in (1) and (2) can be extended by introducing the magnetic force (F_m) into the system dynamic equation

$$M_{eq}\ddot{x}(t)+C_{eq}\dot{x}(t)+K_{eq}x(t)+\Theta_b V(t)+F_m=-\mu M_{eq}\ddot{x}_0(t) \quad (4)$$

$$V(t)/_{R}+C^S\dot{V}(t)-\Theta_f\dot{x}(t)=0 \quad (5)$$

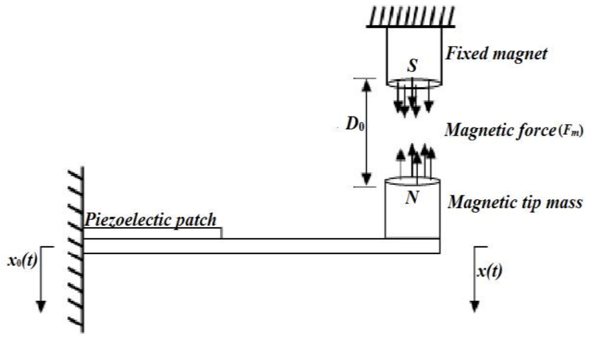

Figure 3 A PEH with magnetic tip mass aligned to a fixed magnet at enclosure.

Where, the magnetic force (F_m) can be established based on the assumptions that there is a dipolar coupling (magnetic dipole-dipole interaction) between the magnets and the magnetic dipoles are always vertically aligned when the PEH vibrates. Based on the aforementioned assumptions, the magnetic force (F_m) between cylindrical magnets is expressed as [15]

$$F_m=\left[\frac{\beta^2A^2(l+r)^2}{\pi\tau_0 l^2}\right]\left[\frac{1}{D_0{}^2}+\frac{1}{(D_0+2l)^2}-\frac{2}{(D_0+l)^2}\right] \quad (6)$$

With τ_0 as the permeability of the medium (air); D_0 is the distance between the magnetic tip mass and the fixed magnet; A is the area of the magnets; l is the height of the magnet; r is the radius and β is the magnetic flux density (magnetic field). When $l \ll D_0$ the approximation of the magnetic force (F_m) on point dipole is reduced to

$$F_m=-\frac{3\tau_0 m_1 m_2}{2\pi[D_0]^4} \quad (7)$$

m_1 *and* m_2 are the moments of the magnetic dipoles, which is mathematically represented as

$$m=\frac{2\beta V}{\tau_0} \quad (8)$$

Where V is the volume of magnet which depends on the geometry of the magnets. With tip displacement $x(t)$ of the PEH system, the magnetic force in equation (7) can be written as

$$F_m=-\frac{3\tau_0 m_1 m_2}{2\pi[x(t)+D_0]^4} \quad (9)$$

The electromechanical coupled governing equation of the system can therefore be written as,

$$M_{eq}\ddot{x}(t)+C_{eq}\dot{x}(t)+K_{eq}x(t)+\Theta_b V(t)-\frac{3\tau_0 m_1 m_2}{2\pi[x(t)+D_0]^4}=-\mu M_{eq}\ddot{x}_0(t) \quad (10)$$

$$V(t)/_{R}+C^S\dot{V}(t)-\Theta_f\dot{x}(t)=0 \quad (11)$$

It is worthmentioning that for a attractive magnets, m_1=-m_2 while m_1=m_2 for repulsive magnets.

C. State space equation

The discrete parameter models depicting the dynamics of the proposed harvesters in (1)(2) and (10)(11) can be represented in state space form as shown in (12) which is without tip magnetic force and (13) with tip magnetic force respectively; By Letting, $z_1(t)=x(t)$,$z_2(t)=\dot{x}(t)$ *and* $z_3(t)=V(t)$ where $z_1(t)$, $z_2(t)$ *and* $z_2(t)$ are the state variables. Therefore,

$$\begin{aligned}\dot{z}_1(t)&=z_2(t)\\ \dot{z}_2(t)&=-\frac{C_{eq}}{M_{eq}}z_2(t)-\frac{K_{eq}}{M_{eq}}z_1(t)-\frac{\Theta_b}{M_{eq}}z_3(t)-\mu\ddot{x}_0(t)\\ \dot{z}_3(t)&=\frac{\Theta_f}{C^S}z_2(t)-\frac{1}{RC^S}z_3(t)\end{aligned} \quad (12)$$

$$\begin{aligned}\dot{z}_1(t)&=z_2(t)\\ \dot{z}_2(t)&=-\frac{C_{eq}}{M_{eq}}z_2(t)-\frac{K_{eq}}{M_{eq}}z_1(t)-\frac{\Theta_b}{M_{eq}}z_3(t)+\frac{3\tau m_1 m_2}{M_{eq}2\pi[z_1(t)+D_0]^4}-\mu\ddot{x}_0(t)\\ \dot{z}_3(t)&=\frac{\Theta_f}{C^S}z_2(t)-\frac{1}{RC^S}z_3(t)\end{aligned} \quad (13)$$

D. Natural frequency tunning with dsitance between magnet

The mathematical equation relating the tuned resonance frequency and distance between magnets is developed in this subsection. As earlier mentioned, the magnetic force (F_m) is used to tune the natural frequency (ω_n) of the PEHs so as to match the excitation frequency, which amounts to resonance phenomena (a desirable property in vibration based energy harvesting) and broadband frequency. In practice, this can be achieved by altering the stiffness of the

PEHs through magnetic force by adjusting the distance between the magnets. For a cylindrical magnets, by adjusting the distance D_0, the magnetic force (F_m) as shown in our proposed harvester in Fig. 3 introduces stiffness unto the system, therefore, change in F_m in (6) with respect to D_0 yield

$$\overline{K}_m = \left|\frac{\partial F_m}{\partial D_0}\right| = \left|\frac{\beta^2 A^2 (l+r)^2}{\pi\tau_0 l^2}\left[-\frac{2}{D_0^3} - \frac{2}{(D_0+2l)^3} + \frac{4}{(D_0+l)^3}\right]\right| \tag{14}$$

Where $\overline{K}_m$ is the magnitude of the stiffness introduced, therefore the effective stiffness of the PEH is given by

$$K_{eff} = K_{eq} + \overline{K}_m \tag{15}$$

Whereas, K_{eq} is the original stiffness of the harvester without the magnetic force. The tuned resonance frequency of the PEH (ω_t) can be obtained by the equation:-

$$\omega_t = \sqrt{\frac{K_{eff}}{M_{eq}}} = \sqrt{\frac{K_{eq} \pm \overline{K}_m}{M_{eq}}} \tag{16}$$

It is worth mentioning that $\overline{K}_m$ can be either negative or positive depending on the nature of magnetic force (attractive or repulsive). A sign convention of (-ve) for attractive is adopted in this paper. By substituting (14) into (15), the relationship between tuned resonance frequency and the distance between magnets experiencing attractive force is given by;

$$\omega_t = \sqrt{\frac{K_{eq} - \left|\frac{\beta^2 A^2 (l+r)^2}{\pi\tau_0 l^2}\left(-\frac{2}{D_0^3} - \frac{2}{(D_0+2l)^3} + \frac{4}{(D_0+l)^3}\right)\right|}{M_{eq}}} \tag{17}$$

III. SIMULATION RESULTS AND DISCUSSION

The Geometrical parameters of the beam and material properties used in the simulation are given in Table 1. It is assumed that the piezoelectric material used is of type Micro Fiber Composite (MFC).

TABLE 1 Parameter Descriptions and their values

Description	**Symbols**	**Value**	**Units**
Aluminum Beam dimension	-	80×10×0.6	*mm*^{3}
Distributed mass of beam	M_b	1.676	*grams*
Young's Modulus of beam	E_b	70×10^9	N/m^2
Damping ratio	ξ	0.0085	-
Tip mass	M_t	3.995	*grams*
Piezoelectric material dimension.	-	42×7×0.3	*mm*^{3}
Clamp capacitance	C^S	12.45×10^{-9}	*Farad*
Electro mechanical coupling coefficient.	$\Theta_b = \Theta_f$	1.496×10^{-4}	$N\ V^{-1}$
Young's Modulus of piezoelectric patch.	E_p	30×10^9	N/m^2
Length of magnet	l	3.0	*mm*
Diameter of magnet	d	4.0	*mm*
Surface flux (magnetic field)	β	1.4	*Tesla*
Distance between magnets	D_0	10	*mm*
Permeability of medium	τ_0	$4\pi\times10^{-7}$	H m^{-1}

The equivalent mass M_{eq} of the proposed cantilever PEH can be estimated by

$$M_{eq} = \frac{33}{140} M_b + M_t \tag{18}$$

Also, the equivalent stiffness can be approximated using the following relation

$$K_{eq} = K_{beam} + K_{patch} = \left(\frac{3E_b I_b}{L_b^3}\right)_{beam} + \left(\frac{3E_p I_p}{L_p^3}\right)_{patch} \tag{19}$$

$$\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}} \tag{20}$$

The equivalent damping of the PEH is calculated using the estimated value of damping ratio by

$$C_{eq} = 2\xi M_{eq}\omega_n \tag{21}$$

The state-space equations developed for the PEHs are used to perform 3-D simulation of power output versus increase in tip mass for a range of frequency using (MATLAB ode45). The output power is computed by

$$P = \frac{V^2 R_{load}}{(R_{source} + R_{load})^2} \tag{22}$$

Where R_{source} and R_{load} are the impedance of PEH and load resistance respectively. For maximum power output of the PEHs, $R_{source} = R_{load}$ a phenomenon known as impedance matching. Therefore an optimal load resistor R_{load} can be estimated by

$$R_{load} = \frac{1}{\omega_n . C^S} \tag{23}$$

A. Power output and frequenc sweep for the PEHs configuration

The 3-D simulation results of power generated from the configurations presented in Fig. 1 and 3 by increasing tip mass (M_t) over a range of frequency from 0-100 Hz is performed on MATLAB software as shown in Fig 4 and 6. The z-x axis dimension of the 3-D simulation which consititute of power on z-axis and frequency sweep on x-axis are presented for analysis. In the simulation for conventional PEH shown in Fig. 5, a maximum power output of 1.88mW is achieved at a frequency of 87Hz. Whereas, for PEH with a single fixed magnet at enclosure, a power output of 1.25mW is produced at the same frequency of 87Hz as shown in Fig. 7. By comparing the power outputs for the different configurations presented, it is observed that conventional PEH yeilds more power output by about 33.5% more than PEH with a fixed magnet at the enclosure of the systems.

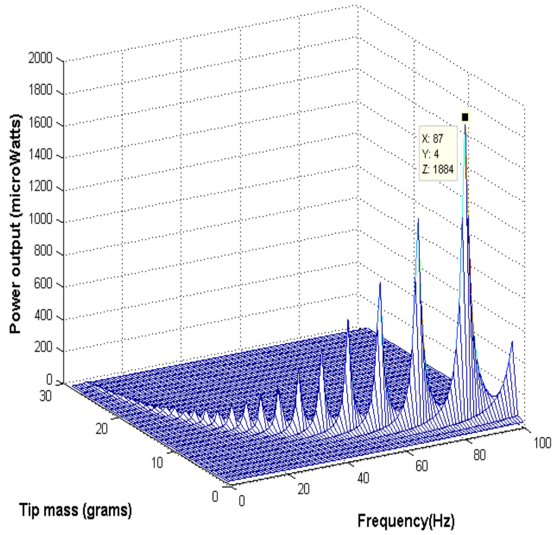

Figure 4 A 3-D simulation of power output versus tip mass and frequency for conventional PEH

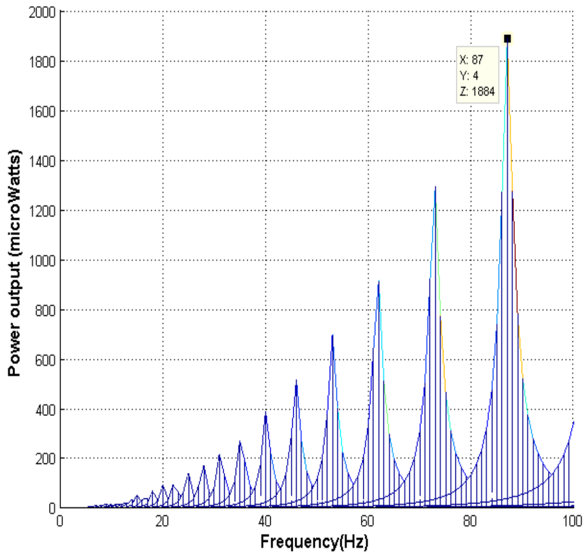

Figure 5 plot f the power versus frequency for conventional PEH

It is evident that the pronounced difference in power output is attributted to damping introduced by the magnetic force which dissipates some of the energy produced by the harvester configuration in Fig 3. Also, it is observed that the non uniformity of the power output in harvester with the magnet is as a result of non linearity introduced by the magnet, making the response slightly tilt to the right as seen in Fig. 7.

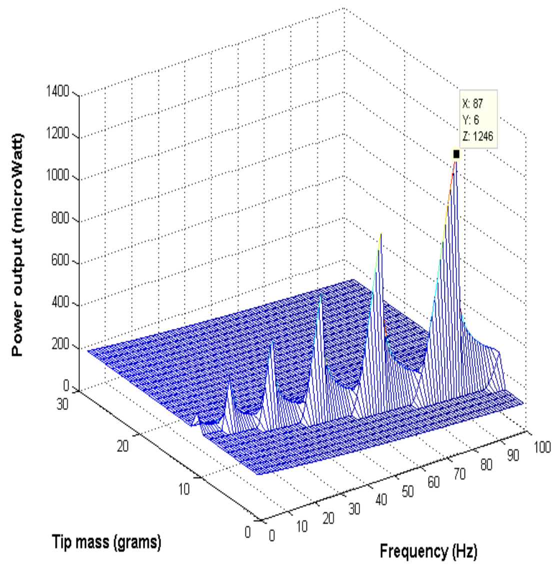

Figure 6 A 3-D simulation of the power output versus tip mass and frequency for PEH with fixed magnet

However, when a wider frequency bandwidth is considered, the later harvesters outperform the conventional PEH.

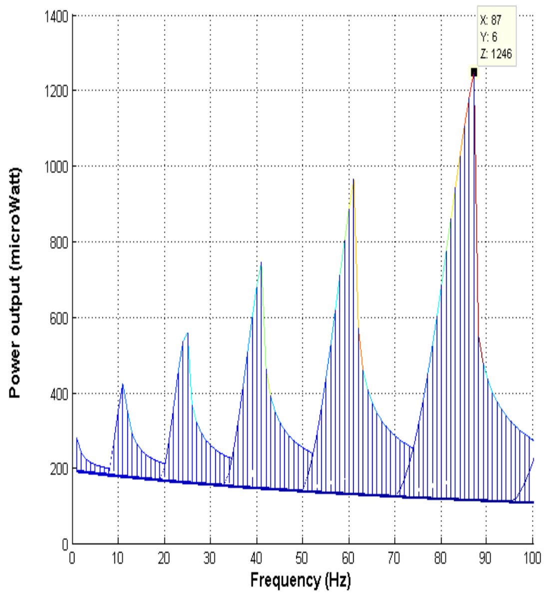

Figure 7 plot of the power output versus frequency for PEH with fixed magnet.

A half power bandwith analysis is performed for this study. The analysis is done by taking ($P_{max}/\sqrt{2}$) of the maximum power output obtained from the simulation plots. A horizontal line is drawn from the point where power equals $P_{max}/\sqrt{2}$ on the plot. Two lines are then projected to the frequency axis forming points of frequency $f_1$and f_2 respectively. The difference between f_1 and f_2 is refered to as *Half power bandwidth.* By analysing the half power bandwidth of the conventional PEH, it is observed that the diffecence in frequency $\Delta f = f_2 - f_1$ is 0.91Hz (less than 1 Hz) which signifies a very narrow power bandwidth. Conversily, for PEH with single fixed magnet, $\Delta f = 5.02Hz$.

It is worthmentioning that the half power bandwidth analysis revealed the fact that when designing a PEH, a trade off needs to be done between high power output and wide frequency bandwidth. However, when a perfect design of PEH is carried out, a balance of relatively high power output and wide frequency bandwidth can be achieved.

B. Tuned resonance frequency and distance between magnets

The application of magnetic force in vibration based energy harvesters is to introduce stiffness in harvesters which in turn tuned the resonace frequency of the PEH, making a device a broadband energy scavenger. By applying an attractive magnetic force as seen in Fig. 3, the natural frequency of the PEH beam can be tuned to lower frequencies with respect to the initial natural frequency of the harvester (ω_n), while the distance required for a given tuned resonance frequency can be obtained by using the plot in Fig 8. The tuned resonance frequency versus distance is plotted in log-linear scale.

We observed that the distance for a specified tuned frequency depends on the magnetic field (β) induced by the magnets as shown in Fig. 7. For lower magnetic field (Example β_1=1.4 Tesla), the distance between the magnets is lower compared to when β_2=2.8 Tesla.

In nut shell, the distance for a given tuned resonance frequency increases with increase in magnetic field. If the distance (D_0) between the magnets is zero, the tuned resonance frequency will approach infinity, meaning that the magnets snaped making the PEH extremely stiff. As D_0 increases, the stiffness introduced by magnetic force on the harvester decreases upto a point where there will be no influence of magnetic force on the stiffness of the system.

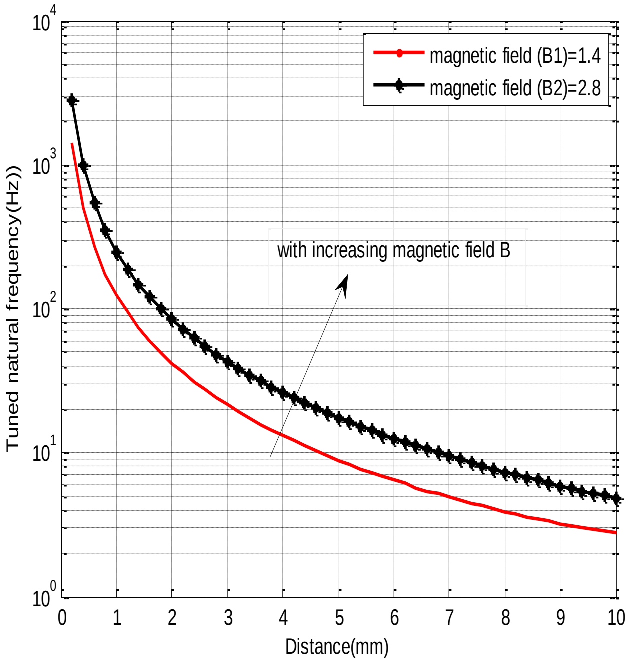

Figure 8 Plot of tuned resonance frequency versus distance between magnets in Log - scale

IV. CONCLUSION

This paper presents the mathematical model of single fixed magnet in an enclosure of piezoelectric energy harvester system. The simulation studies of power output over a range of frequency are performed. We observed that magnetic force undermines the power output of the piezoelectric energy harvester systems. However, when wider frequency bandwidth is required, Linear PEH has a very narrow bandwidth compared to harvester with magnetic influence. Therefore, in designing a PEH, a balance should be aimed between high power output and wide frequency bandwidth. Mathematical equation relating the tuned resonance frequency and the distance between magnets is established. We also realized that the distance required for tuning is directly proportional to magnetic field induced by the magnets. This could be an important design relationship when developing a broadband vibration energy harvester.

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