Comparison of Quadrotor Performance Using Backstepping and Sliding Mode Control

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Abstract: A quadrotor is nonlinear, coupled and unstable system. Two control schemes, namely backstepping and sliding mode, have been applied to obtain desired trajectory tracking by quadrotor. This paper presents the comparative performance results of quadrotor under two control schemes.

Keywords: Micro Quadrotor; Backstepping Control; Sliding Mode Control

I. INTRODUCTION

There are numerous applications of Unmanned Aerial Vehicles (UAVs) in defence and civil areas for monitoring, remote sensing, surveillance, dangerous environment etc. Quadrotor helicopter is an emerging rotor craft concept for UAV that consist of four rotors, with two pair of counter rotating, fixed pitch blades located at the four corners of the aircraft. A quadrotor is a dynamic vehicle with four input forces, six output coordinators, highly coupled and unstable dynamics [1-2]. Hence the design of a control law is an interesting challenge. Several linear methods, such as PID and LQR control method have been applied to control a quadrotor [3-4]. Since the quadrotor is a nonlinear system and for a good performance the nonlinear control methods have been attempted such as feedback linearization, sliding mode and backstepping control [9-11]. This paper presents two nonlinear control techniques applied to a micro quadrotor for developing a reliable control system for stabilization and trajectory tracking.

Nonlinear backstepping control technique forces the system to follow the desired trajectory [5-8]. A backstepping control algorithm was proposed [7] to stabilize the whole system and able to drive a quadrotor to the desired trajectory of Cartesian position and yaw angle. The backstepping control has been modified [9-11] to reduce the control parameters by half compared with the classical backstepping approach as given in [9]. A sliding mode controller was developed to ensure Lyapunov stability, and follow the desired trajectories [18].

The dynamical model of micro quadrotor is presented in section II. Backstepping control is explained in section III. Section IV describes the sliding mode control. The simulation results for quadrotor performance have been presented in section V. The control performance comparison is discussed in section VI.

II. DYNAMICAL MODEL OF QUADROTOR

The full order Quadrotor dynamical modelling has been presented in [5-11]. A quadrotor is an under actuated aircraft with fixed pitch angle four rotors as shown in fig. 1. These four rotors represent four input forces that are basically the thrust generated by each propeller. The collective input \( u_1 \) is the sum of the thrusts of each motor. Pitch moment is obtained by increasing (reducing) the speed of the rear motor while reducing (increasing) the speed of the front motor. The roll movement is obtained similarly by increasing (reducing) the speed of the right motor while reducing (increasing) the speed of the left motor. The yaw movement is obtained by increasing (decreasing) the speed of the front and rear motors together while decreasing (increasing) the speed of the lateral motors together. The outputs of the system are \( x, y, \) and \( z \), which denote the position of the vehicle with respect to the earth frame, and \( p, \) \( q \) and \( r \), which denote the angular velocity of the vehicle with respect to body frame.

The dynamic model is derived using Euler-Lagrange formalism [1], [5], [13]. The equations describing the dynamics of the micro quadrotor are [5],

\[
\begin{align*}
\dot{\phi} &= \dot{\theta} \frac{(L_x - L_z)}{I_x} - \frac{J_z}{I_x} \theta \Omega + \frac{1}{I_x} U_2 \\
\dot{\theta} &= \dot{\phi} \frac{(L_x - L_z)}{I_y} - \frac{J_z}{I_y} \phi \Omega + \frac{1}{I_y} U_3 \\
\dot{\phi} &= \phi \dot{\theta} \frac{(L_x - L_z)}{I_x} + \frac{1}{I_x} U_4 \\
\ddot{z} &= -g + (\cos \theta \cos \phi) \frac{1}{m} U_1 \\
\ddot{x} &= (\cos \phi \sin \theta \cos \phi + \sin \theta \sin \phi) \frac{1}{m} U_1 \\
\ddot{y} &= (\cos \phi \sin \theta \sin \phi - \sin \phi \cos \phi) \frac{1}{m} U_1
\end{align*}
\]

Fig.1:- Scheme of quadrotor helicopter

The first term in the orientation subsystem \( (\dot{\phi}, \dot{\theta}, \phi) \) is the gyroscopic effect resulting from the rigid body rotation in space and the second one is due to the propulsion group rotation. The system’s input are expressed as \( U_1, U_2, U_3, U_4 \) and \( \Omega \) a disturbance, obtaining:
\[ u_1 = b (\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \]
\[ u_2 = b (\Omega_2^2 - \Omega_1^2) \]
\[ u_3 = b (\Omega_3^2 - \Omega_1^2) \]
\[ u_4 = d (\Omega_2^2 + \Omega_3^2 - \Omega_1^2 - \Omega_4^2) \]
\[ \Omega = \dot{r}_2 + \dot{n}_4 - \dot{n}_1 - \dot{n}_3 \]

The dynamic model presented in equation set (1) can be written in state space form as, 
\[ \dot{X} = f(X, U) \] by introducing state vector as 
\[ X^T = [\theta, \dot{\theta}, x, \dot{x}, y, \dot{y}] \]
\[ x_1 = \theta, \quad x_2 = \dot{\theta}, \quad x_3 = x, \quad x_4 = \dot{x}, \quad x_5 = \phi, \quad x_6 = \dot{\phi}, \quad x_7 = z, \quad x_8 = \dot{z}, \quad x_9 = y, \quad x_{10} = \dot{y} \]

From (1) and (2) we obtained
\[ \dot{X} = \begin{pmatrix} x_2 \\ x_4 x_6 a_1 + x_4 a_2 \Omega + b_1 U_2 \\ x_4 \\ x_2 x_6 a_3 + x_2 a_4 \Omega + b_2 U_3 \\ x_6 \\ x_4 x_6 a_5 + b_3 U_4 \\ x_8 \\ -g + (\cos x_1 \cos x_3) \frac{1}{m} u_1 \\ x_{10} \\ \frac{1}{m} u_1 \\ x_{12} \\ \frac{1}{m} u_1 \end{pmatrix} \] .......(3)


The overall system described by (3) has two subsystems, the angular rotations and the linear translations [2]. The control scheme for the overall system is then logically divided in a position controller and a rotational controller.

### III BACKSTEPPING CONTROL

In backstepping approach, the control law is synthesized to force the system to follow the desired trajectory [5-8]. Due to its complete independence from the other subsystem, firstly consider the control input for angular rotations subsystem and then the position control input is derived. The tracking error is defined as 
\[ z_1 = x_{1d} - x_1 \] , and the Lyapunov function
\[ V(z_1) = \frac{1}{2} z_1^2 \]

Therefore
\[ \dot{V}(z_1) = z_1 (\dot{x}_{1d} - \dot{x}_1) \]

Introducing the virtual control input \( x_2 \) for stabilization of \( z_1 \)
\[ x_2 = x_{1d} + a_1 z_1 \]

Then \( V(z_1) = -a_1 z_1^2 \)

Making the variable change
\[ z_2 = x_2 - \dot{x}_{1d} - a_1 z_1 \]
\[ V(z_1, z_2) = \frac{1}{2} (z_1^2 + z_2^2) \]

It follows,
\[ \dot{V}(z_1, z_2) = z_2 (a_1 x_{1d} + a_2 x_2 + b_1 U_2) - z_2 (x_{1d} - a_1 z_1 - a_2 z_1) \]

\[ \dot{x}_{1d,2,3d} = 0 \] satisfying \( V(z_1, z_2) < 0 \)

So \( U_2 \) can be extracted as
\[ U_2 = \frac{1}{b_3} (z_1 - a_1 x_4 x_6 - a_2 x_4 \Omega - a_3 (z_2 + a_1 z_1) - a_2 z_2) \]

\( U_3, U_4 \) and \( U_1 \) can be calculated with the same procedure
\[ U_3 = \frac{1}{b_3} (z_5 - a_3 x_2 x_6 - a_4 x_2 \Omega - a_3 (z_4 + a_3 z_3) - a_4 z_4) \]
\[ U_4 = \frac{1}{b_3} (z_5 - a_5 x_2 x_4 - a_6 (z_2 + a_1 z_1) - a_6 z_6) \]

and control input for translation subsystem is [4],
\[ U_1 = \frac{m}{\cos x_1 \cos x_3} (x_7 + g - a_7 (x_6 + a_7 z_2) - a_6 z_6) \]
\[ u_x = \frac{m}{U_1} (x_{11} - a_{11} (x_{12} + a_{11} z_{11}) - a_{12} z_{12}) \] ......... (4)

where
\[ z_3 = x_3d - x_3 \]
\[ z_4 = x_4 - \dot{x}_{3d} - a_3 z_3 \]
\[ z_5 = x_5d - x_5 \]
\[ z_6 = x_6 - \dot{x}_{5d} - a_5 z_5 \]
\[ z_7 = x_7d - x_7 \]
\[ z_8 = x_8 - \dot{x}_{7d} - a_7 z_7 \]
\[ z_9 = x_{9d} - x_9 \]
\[ z_{10} = x_{10} - \dot{x}_{9d} - a_9 z_9 \]
\[ z_{11} = x_{11d} - x_{11} \]
\[ z_{12} = x_{12} - \dot{x}_{11d} - a_{11} z_{11} \]

### IV SLIDING MODE CONTROL

The basic sliding mode controller design procedure is performed in two steps. Firstly, choice of sliding surface (S) is made according to the tracking error, while the second step consist the design of Lyapunov function which can satisfy the necessary sliding condition \( S \dot{S} < 0 \)[9-11]. The

### Table 1.1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>Mass</td>
<td>650kg</td>
</tr>
<tr>
<td>I_x</td>
<td>Inertia on x axis</td>
<td>7.5e-3 kgm^2</td>
</tr>
<tr>
<td>I_y</td>
<td>Inertia on y axis</td>
<td>7.5e-3 kgm^2</td>
</tr>
<tr>
<td>I_z</td>
<td>Inertia on z axis</td>
<td>1.3e-2 kgm^2</td>
</tr>
<tr>
<td>b</td>
<td>Thrust coefficient</td>
<td>3.13e-5 Ns^2</td>
</tr>
<tr>
<td>d</td>
<td>Drag coefficient</td>
<td>7.5e-7 Nms^2</td>
</tr>
<tr>
<td>J_r</td>
<td>Rotor inertia</td>
<td>6e-5 kgm^2</td>
</tr>
<tr>
<td>L</td>
<td>Arm length</td>
<td>23 m</td>
</tr>
</tbody>
</table>
application of sliding mode control to quadrotor dynamic is presented here by obtaining the expression for control input. The sliding surface are define,

\[
\begin{align*}
S_\theta &= e_2 + \lambda_1 e_1 \\
S_\phi &= e_4 + \lambda_2 e_3 \\
S_\psi &= e_6 + \lambda_3 e_5 \\
S_x &= e_8 + \lambda_4 e_7 \\
S_y &= e_{10} + \lambda_5 e_9 \\
S_z &= e_{12} + \lambda_6 e_{11}
\end{align*}
\]

Such that \( \lambda_i > 0 \) and \( e_i = x_{id} - x_i e_{i+1} = e_{i} \ i \in [1,11] \)

Assuming here that \( V(S_\theta) = \frac{1}{2} \dot{S}_\theta^2 \) then, the necessary sliding condition is verified and Lyapunov stability is guaranteed. The chosen law for the attractive surface is the time derivative of (21) satisfying \( (S_\theta, S_\phi) < 0 \)

\[
\begin{align*}
\dot{S}_\theta &= k_1 \text{sign}(S_\theta) \\
&= \dot{e}_2 + \lambda_1 e_1 \\
&= \dot{x}_{1d} - \dot{x}_2 + \lambda_1 (x_{1d} - x_2) \\
&= -x_4 x_4 a_1 - x_4 a_4 \omega - a_1 U_2 + \dot{\phi}_d + \lambda_1 (\dot{\phi}_d - x_2) \\
U_2 &= \frac{1}{b_1} \left( - k_1 \text{sign}(S_\theta) - a_1 x_4 x_6 - x_4 a_2 \omega + \dot{\phi}_d + \lambda_1 (\dot{\phi}_d - x_2) \right) \\
&= \frac{1}{b_1} \left( - k_1 \text{sign}(S_\theta) - a_1 x_4 x_6 - x_4 a_4 \omega + \dot{\phi}_d + \lambda_1 e_2 \right)
\end{align*}
\]

The same steps are followed to extract \( U_3, U_4, U_5, U_6, U_7 \)

\[
\begin{align*}
U_3 &= \frac{1}{b_2} \left( - k_2 \text{sign}(S_\theta) - a_3 x_2 x_6 - x_2 a_4 \omega + \dot{\phi}_d + \lambda_2 e_4 \right) \\
U_4 &= \frac{1}{b_3} \left( - k_3 \text{sign}(S_\psi) - a_5 x_3 x_6 - x_3 a_4 \omega + \dot{\phi}_d + \lambda_3 e_6 \right) \\
U_5 &= \frac{m}{\cos x_3 \cos x_2} \left( - k_4 \text{sign}(S_\psi) + g + \dot{z}_d + \lambda_4 e_8 \right) \\
U_6 &= \frac{m}{U_1} \left( - k_5 \text{sign}(S_\theta) + \dot{x}_d + \lambda_5 e_{10} \right) \\
U_7 &= \frac{m}{U_1} \left( - k_6 \text{sign}(S_\psi) + \dot{y}_d + \lambda_6 e_{12} \right)
\end{align*}
\]

\[
\begin{align*}
\text{IV SIMULATION RESULTS}
\end{align*}
\]

Two nonlinear control techniques i.e. “Backstepping” and “Sliding Mode Control” have been exercised on nonlinear model of micro quadrotor to demonstrate the control performance. Several simulations are done in Matlab using model (3) with 12 parameters \( (a_1, a_2, \ldots, a_{12}) \) controller and to reach the position \( x_4 = y_4 = z_4 = 2m \) with the initial condition \( \pi \) rad for the three angles.

a) Backstepping control: The control inputs as derived equation (4) have been implemented to the nonlinear model in (3) and responses shown in fig. (3). Simulation have been performed for a desired position tracking from \((0, 0, 0)\) to \((2, 2, 2)\) given in fig. (2).

b) Sliding Mode Control: The sliding mode control inputs which were derived and expressed in equation (5) were applied to the nonlinear model in (3) and responses are shown in fig. (4). Quadrotor dynamics is stabilised following the given position.

\[
\begin{align*}
\text{VI. CONCLUSION}
\end{align*}
\]

This paper has considered two nonlinear control techniques (a. Backstepping Control, b. Sliding Mode Control) and a nonlinear unstable system, quadrotor which has several applications. The control equations have been derived for quadrotor dynamics. The control implementation has been exercised through simulation in MATLAB. The results have been presented here. Both the schemes stabilize the quadrotor as they are based on Lyapunov theory. Both the
schemes are robust up to 10% change in parameters. However performance with sliding mode control is smooth and faster. This study has motivated to modify these schemes for further improvement in performance which is under study.

References


