The prediction of mortality by causes of death in Critical Illness

Valeria D'Amato[1], Vincenzo Passannante[2] , Marilena Sibillo[3]

***Abstract*—** The paper proposes a new contractual structure built within the critical illness policy model. The new product is represented by an accelerated critical illness with a special accelerated benefit in case of death for a specific cause. The inclusion of the benefit in case of a specific cause death does not involve additional cost to the life office beyond the critical illness benefit. On the contrary the new design ensures less expensive conditions in comparison with the standard policy and it is appealing from the market point of view, looking for more and more personalized clauses.

We propose a novel form for modelling framework of the product under consideration. In order to highlight the profiles of commercial attractiveness of the product, we measure the premium price of the specific accelerated critical illness, which will be illustrated for different ages and compared to a standard accelerated product. Furthermore, actuarial valuations are performed for indicating how much money is needed to fulfil the obligations of the insurance contracts defining premiums to be received and benefits to be paid..

***Keywords*—** Causes of death, Lee-Carter, Critical Illness, Insurance, forecasting.

I. Introduction

The study of cause-specific mortality time series is one of the main sources of information for public health monitoring. Models for trends in mortality rates for different ages and sexes as well as for different countries are often based on the assumption that past trends in historical data will continue in the future. Mortality trends and related fluctuations determine changes in the causes of deaths. These causes have different age patterns and have shown different trends over recent years. At the same time, systematic changes in causes of death have been common across the industrialized economies.

Recent literature has addressed the issue of cause-specific mortality analysis. In particular, Maccheroni et al. (2007) examine how the Lee-Carter model is not suitable for the analysis by causes of death. Sherris et. al. (2010) discuss the factors driving mortality changes based on causes of death. Tuljapurkar et al. (2000) show how mortality declines have had common trends in the G7 countries, although there is evidence of variability in those trends. Booth et al. (2006) also demonstrate the difficulties related to the projections obtained by the decomposition of the population according to causes of death. Wilmoth (1995) shows how taking into account causes of death can influence projected trends and effectively highlights how cause of death influence is hidden in aggregated data.

The World Health Organization (World Health Organization, 2009) has revised the international classification of diseases (ICD) approximately every 10 years since 1900. The purpose of revision is to stay abreast of advances in medical sciences, changes in medical terminology and to ensure the international comparability of health statistics. However, the ICD revision often causes major discontinuities in trends of mortality and morbidity statistics because of changes in classification rules for selecting underlying causes of death. The ranking of leading causes of death is also affected by this revision. These discontinuities lead not only to a misinterpretation of trends in mortality, but also to misinformation about the changes in life expectancy (Kochanek et al., 1994). Furthermore, without properly correcting these discontinuities, trends in age-specific death-rates may become distorted; this distortion may lead to unreliable forecasts of life expectancy. The problem induced by the study of cause-specific mortality, related to the jumps caused by the reclassification ICD, can be mitigated by the model of Haberman et al. (2013).

The Critical Illness Insurance market, in particular in UK, growth up to 1999, was followed by a plateau in 2000 and 2001. Sales peaked in 2002, when over one million accelerated critical illness policies were sold (CMI, 2010). These policies pay an assurance benefit on the occurrence of a serious event, such as the diagnosis of an illness. Most are sold as 'accelerated benefits' riders with life insurance policies. In this context reliable projections of survival probabilities are crucial to correctly determine insurance premiums, technical provisions and other actuarial valuations.

The contribution of this work is to propose an original insurance policy, in the market of the Critical Illness, designing a contract which includes a benefit in case of specific death-cause, beyond the benefit in case of specified illness for a policyholder more and more demanding.

We introduce a modelling framework of the product under consideration and price the accelerated benfit by forecasting survival probabilities throughout tables for specific death-causes.

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The layout of the paper is the following: in section 2, we introduce the novel critical illness coverage and a mark-to-market valuation framework. Section 3 describes a specific death-cause mortality model we use for projecting specific life tables. Numerical applications are illustrated in section 4. Concluding remarks are offered in section 5.

II. Critical Illness Cover with a Specific Accelerated Benefit

The basic critical illness policy is a very simple product. Normally, you have a lump sum cash payment that's paid upon the occurrence or diagnosis of one of a number of specified diseases or conditions. There are some conditions for setting the coverage. A benefit is paid if the assured suffers or dies for one the following critical conditions (Heart Attack, Coronary Artery Bypass Surgery, Stroke, Cancer, Major Head Trauma, Severe Burns

There are two main contractual options: the Standalone Cover and the Accelerated Cover.

The acceleration is the coverage in case of two events, one relating to the risk of falling ill and the other relating to the risk of dying.

Actuarial models for Dread Disease (DD) insurance can be built up starting from a multistate structure. In Fig. 5 the following states are considered:

$\alpha =$ healthy;
$i =$ ill, dread disease suffer;
$d(D) =$ dead being due to dread disease;
$d(O) =$ deaths being due to other causes
$\lambda =$ the portion payable on DD diagnosis
$1-\lambda =$ the portion payable on death

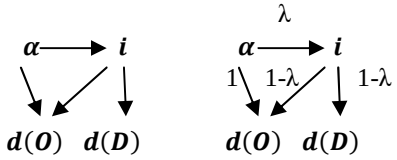

Fig. 1 Multistate schemes for DD insurance.

The following intensities define the time-continues probabilistic structure:

$\mu_y^{ai} =$ intensity of transition from a to i;

$\mu_y^{ad(O)} =$ intensity of transition from a to d(O);

$\mu_{y,r}^{id(O)} =$ intensity of transition from i to d(O);

$\mu_{y,r}^{id(D)} =$ intensity of transition from i to d(D);

The mortality of DD suffers is concerned, the calculation of actuarial values only requires the use of the total force of mortality $\mu^{id(O)} + \mu^{id(D)}$, since benefits are usually independent on the cause of death. The probabilities of interest for actuarial calculations are:

$$ {}_tp_y^{aa} = exp\left[-\int_0^t (\mu_{y+u}^{ai} + \mu_{y+u}^{ad(O)})du\right] \quad (1)$$

$$ {}_\tau p_{y+u,0}^{ii} = exp\left[-\int_0^t (\mu_{y+u+r,r}^{id(O)} + \mu_{y+u+r,r}^{id(D)})dr\right] \quad (2)$$

The attained age is denoted by y and r denotes the time elapsed since DD inception.

Let us consider a temporary assurance with a DD acceleration benefit. We assume that, for a given sum S, the amount λS (where $0 < \lambda \leq 1$, the acceleration parameter) is payable on DD diagnosis, while the remaining $(1-\lambda)S$ is payable on a specific cause of death if this occurs within the policy term *n*. In our case we let ${}_hq_{x,c}^{aa}$ denote the probability of death at time h for c cause and for simplicity we restrict our attention to an acceleration benefit with $\lambda = 1$.

If we calculate the single premium meeting the death benefit as well as the DD acceleration benefit, we have:

$$ A_{x,n}^{D+DD;1} = \sum_{h=0}^{n-1} {}_hp_x^{aa}(q_{x+h,c}^{aa} + w_{x+h})v^{h+1/2} \quad (3)$$

where:

- w_x is the probability of becoming a DD sufferer within one year
- ${}_hp_x^{aa}$ is the probability of being healthy at age x+1
- $v^{h+1/2}$ is a discounted factor
-

The relevant annual premium is given by:

$$ P_{x:n|} = \frac{A_{x,n}^{D+DD;1}}{\ddot{a}_{x:n|}^{aa}} \quad (4)$$

In particular the value of the reserve is given by:

$$ V_{x+t,n-t}^{a} = V_{x+t,n-t}^{(D+DD;1)} - P_{x:n|}\ddot{a}_{x+t:n-t|}^{aa} \quad (5)$$

Let us indicate by k_x the healthy curtate future lifetime of the insured aged x at issue. The cash flow scheme related to the policy at time s is the following, in the case of anticipated annual premiums:

$$ X_s = \begin{cases} {}_{/m}P_{x,s+1} & k_x \geq s \quad 0 \leq s \leq m-1 \\ 0 & k_x \geq s \quad s \geq m \\ B_s = F & s-1 \leq k_x \leq s \quad 1 \leq s \leq n \end{cases} $$

in which $s = 1,2,\ldots,n$, $P_{n+1} = 0$, $X_0 = P_1$ and where ${}_{/m}P_{x,s+1}$ is the premium amount payable up to time m and B_s is the critical illness benefit equal to the face value F. In light

of a fair valuation of the policy (Coppola et al. 2009), the stochastic flow of the portfolio f_s at times$, s > t$ by a trading strategy can be expressed as follows:

$$f_0 = -c_{/m}P_{x,1} \qquad \text{if } s = 0$$

$$f_s - {}_{/m}P_{x,s+1}y_s + B_s y_s^u + B_s(Y_s - Y_{s-1}) \text{ if } s = 1,2,\dots,n$$

where y_s is the number of healthy assured among the survivors (briefly healthy survivors) at time s;
Y_s^u is the number of unhealthy assured among the survivors (briefly unhealthy survivors) at time s, in particular having $Y_s^u = Y_s - Y_{s-1}$, with Y_s the number of the healthy and unhealthy survivors at time s.
We formulate the stochastic provision at time t in its fair value form, replicating the stochastic flow F_s at time s as in the following equations:

$$V_t = E\{L_t/I_t\} = E\left\{\sum_{s=t+2}^{n}\left[-{}_{/m}P_{x,s+1}y_s + B_s y_s^u + B_s(Y_s - Y_{s-1})\right]v(t,s)/It\right. \quad (6)$$

where L_t is the stochastic loss in t of the portfolio of c contracts in-force and I_t is represented by the filtration $\{I_t\} \subset I$ containing the information flow at time t . On the basis of the conditional expectation calculus, we can write:

$$V_t = \left\{\sum_{s=t+2}^{n}\left[-{}_{/m}P_{x,s+1}c\ {}_tp_{x\,s-t}p_{x+t} + B_s c q_x^H\right]v(t,s)/ItEv(t,s)/It\right. \quad (7)$$

where ${}_tP_x$is the survival probability of assured aged x up to time t, q_x^h is the rate of the $h-th$ death or diagnosis of a critical illness, referred to an assured aged x, whichever occurs first and v(t,s) the stochastic present value at time t of one monetary unit at time s.

III. Lee-Carter model with cause of death coding adjustments

Assume that the number of deaths are independent Poisson responses $D_{xt} \sim Poisson(e_{xt}\mu_{xt})$. Let $S = \{s_1, s_2 \dots, s_h\}$ be the times at which coding changes occur. In order to account for the coding changes, we assume as in Haberman et al., 2013, that the force of mortality is given by:

$$log_{\mu_{xt}} = \alpha_x + \beta_x k_t + \sum_{i=1}^{h}\delta_x^{(i)}f^{(i)}(t) \quad (8)$$

where:

- μ_{xt} is the age-specific death rate for the *x* interval and the year *t*.
- α_x is the average age-specific mortality.
- k_t is the mortality index in the year t.
- β_x is a deviation in mortality due to changes in the k_t index.
- $f^{(i)}(t) = I_{s_{i-1}} \le t < s_i$ is an indicator function.
- $\delta_x^{(i)}$ measures the magnitude of coding change at age x.

This model assumes that there are different age-patterns $\alpha_x + \delta_x^{(i)}$for each period $[s_{i-1}, s_i)$ where different causes of death coding system is used. The parameters in (8) can be estimated by maximizing the Poisson log-likelihood of the model. The model is over parameterized since the structure is invariant under either the parameter transformations, that is for any constants $b_1, b_2 \neq 0$:

$$\{\widetilde{\alpha_x}, \widetilde{k_t}\} = \{\alpha_x + b_1\beta_x, k_t - b_1\} \quad (9)$$

$$\{\widetilde{\beta_x}, \widetilde{k_t}\} = \left\{\frac{1}{b_2}\beta_x, b_2 k_t\right\} \quad (10)$$

$$\left\{\widetilde{\delta_x^{(i)}}, k_t\right\} = \left\{\delta_x^{(i)} + a_i\beta_x, k_t - a_i f^{(i)}(t)\right\}, \quad i = 1,\dots,h \quad (11)$$

Transformation (9) and (10) are the original ones from the Lee-Carter model, whilst the family of transformation defined by (11) are induced by the new parameters $\delta_x^{(i)}$ (Haberman et al., 2013).
In order to ensure the complete characterization of the model, the following constraints need to be imposed:

$$k_{t_n} = 0 \quad (12)$$

$$\sum_x \beta_x = 1 \quad (13)$$

In the model the underlying mortality trend is captured only by k_t whilst parameters $\delta_x^{(i)}$ capture the discontinuities in mortality trend induced by the changes in the coding system of the causes of death. In order to accomplish this, we use the family of transformation defined in formula (11).
Inspired by the procedure introduced by Ray et al. (2011), we set the constants $a_i, i = 1,\dots,h$, by fitting the model

$$k_t = g(t) + \sum_{i=1}^{h}\delta_x^{(i)}f^{(i)}(t) + \epsilon_t \quad (14)$$

where $g(t)$ is a continuous function fitted by a thin plate penalized regression spline and ϵ_t is an error term. Model (14) decomposes the time trend k_t into:

- a smooth function $g(t)$ representing the underlying mortality trend;
- the jumps in mortality $\sum_{i=1}^{h} a_i f^{(i)}(t)$ induced by data production changes
- the noise ϵ_t around the underlying trend.

The smoothes parameter of $g(t)$ is derived using generalized cross-validation.
Given constants $a_i, i = 1,\dots,h$ from model (14), respectively for k_t and $\delta_x^{(i)}$, are given by:

$$k_t \longrightarrow k_t - \sum_{i=1}^{h} a_i f^{(i)}(t) \quad (15)$$

$$\delta_x^{(i)} \longrightarrow \delta_x^{(i)}f^{(i)} + a_i\beta_x \qquad i = 1,\dots,h \quad (16)$$

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which transfers the jumps in mortality due to data production changes to the $\delta_x^{(i)}$ parameters while k_t represent the underlying mortality trend plus the fluctuations around this trend.

IV. Numerical Application

In order to highlight the profiles of commercial attractiveness of the product, we measure the premium price of the specific accelerated critical illness, which will be illustrated for different ages and compared to a standard accelerated product.
First we calculate mortality rates as the number of persons for each age (x), sex (s), and country (c) who die in a particular year (t) of a specified cause (d), divided by the number of persons of that age and sex in the country alive at the beginning of the year.

$$\mu_{xt} = \frac{D_{x,s,c,d,t}}{E_{x,s,c,d,t}} \tag{17}$$

Data were obtained from the Mortality Database administered by the World Health Organization [2009] (WHO) which contains demographic information, including the number of deaths according to the underlying cause of death, for Italy over the last 50 years from 30 to 89 age.
Causes of death are defined by the International Classification of Diseases (ICD), which ensures consistencies between countries. In this study, only the primary causes of death are consider to be used for the construction of the new policy that we intend to propose.The ICD changed three times between 1950 and 2006, from ICD-7 to ICD-10, in order to take into account changes in science and technology and to refine the classification. In Italy there were two reclassifications (ICD7-8 in 1968, ICD8-9 in 1979) The six main causes of death are diseases of the circulatory system, cancer, diseases of the respiratory system, external causes, infectious and parasitic diseases and other.
In Figure 2 it is possible to observe the fitted parameters and in Figure 3 the fitting trend of the mortality index k_t respect to that observed (that has two jumps in the years in which the ICD changed); both the graphs are referred to the cause of death related to the circulatory system.. Figures 4 and 5 are referred to the cause of death referred to the respiratory system.
With these graphs we can see how the new transformation transfers the jumps in mortality due to data production changes to the $\delta_x^{(i)}$ parameters and leaves k_t representing the underlying mortality trend plus the fluctuations around this trend.
In the plots there are represented the evaluation of the parameters of the model (8). The model eliminates the discontinuities in the mortality rates.

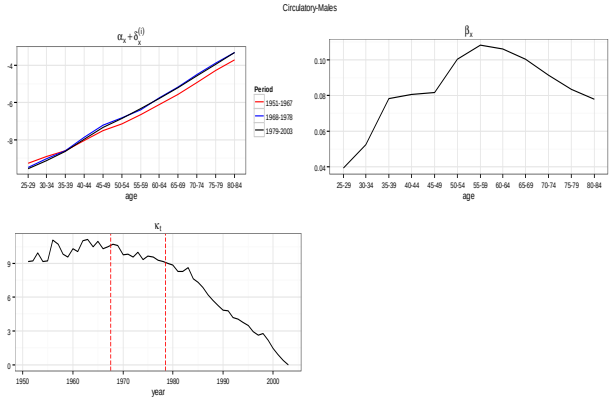

Fig. 2 Multistate schemes for DD insurance.

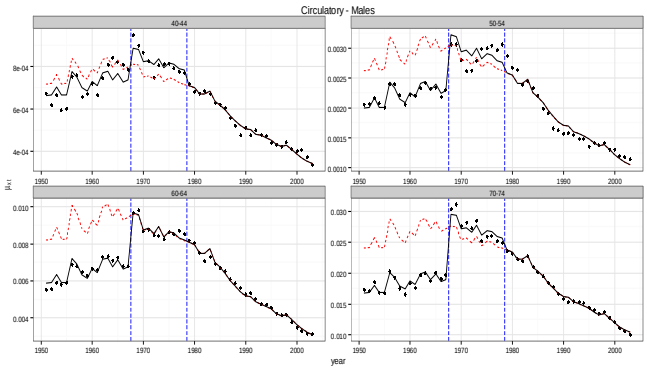

Fig. 3:k_t with the coding changes for different group age

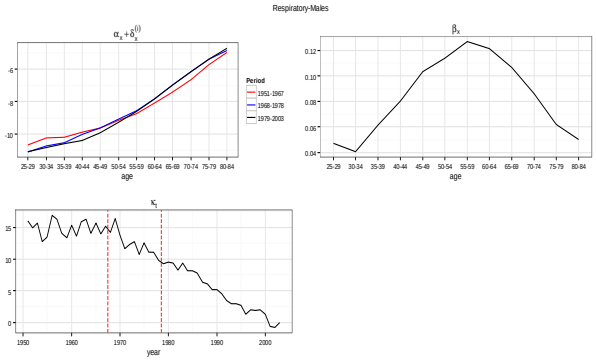

Fig. 4: Fitting parameters Respiratory System

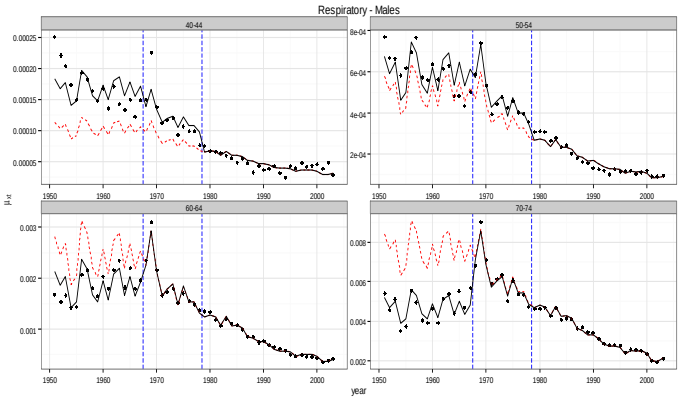

Fig. 5: k_t with the coding changes for different group age

In order to project the age-specific death rates, the model assumes the constancy of α_x, β_x and $\delta_x^{(i)}$. The only parameter to be projected through a procedure Box - Jenkis that serves to determine an appropriate ARIMA is k_t.
According to the model, the mortality rate is a linear trend on the basis of an ARIMA (0,1,0), which is well adapted to the representation of the evolution of the index over time. It therefore refers to the following model (Lee and Carter, 1992):

$$k_t = k_{t-1} - c - e_t \quad (18)$$

In the next plots we will see the performance of the insurance premium for different Critical Illness cover, a Stand-Alone, a Stand-alone Accelereted (with an accelerated benefit for death) and a Stand Alone with a benefit for specific cause of death.

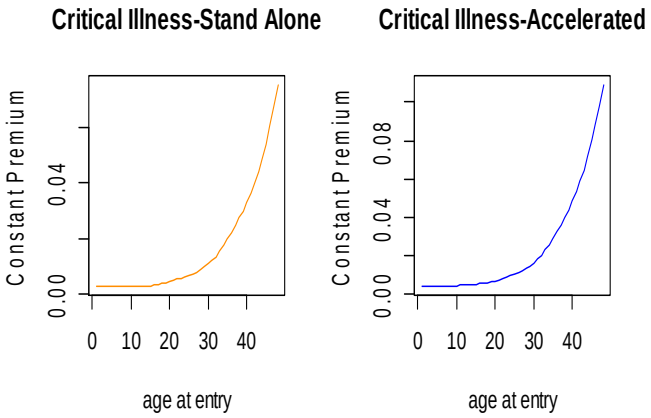

Fig. 6: Performance of the periodic premium for a TCM (Standalone and Accelerated Benefit), term 20 years

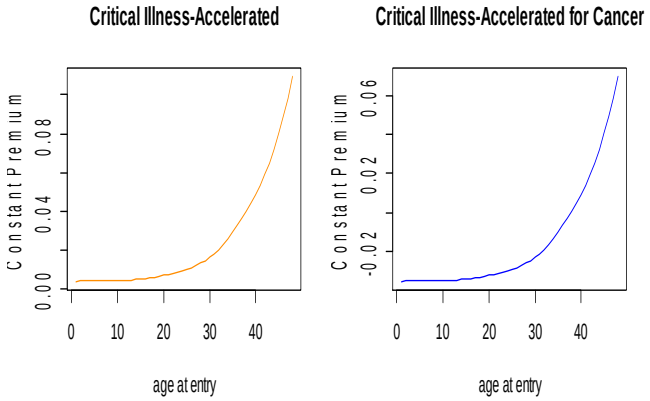

Fig. 7: Performance of the periodic premium for a TCM (Accelerated Benefit and Accelerated Benefit for Cancer), term 20 years

As you can see from the graphs, the premium for the policy that we propose in this work grows slower than the other. This means that it is less expensive than the Standard Stand Alone. It could be more palatable and accessible to families with middle and low incomes.

V. Concluding Remarks

Actuarial services are offered in several forms, more or less structured and complex, according to models in continuous transformation to the aim of matching as much as possible the need of coverage of each potential insured. The insurance companies are strongly interested in designing new products in which the individual can recognize his own characteristics and as a consequence contracts drafted according to generic profiles reveal to be unfit and poorly attractive. This tendency is going to strengthen the use of personalized models in actuarial valuations and in particular of the death probabilities disaggregated by cause of death. The paper considers this topic in the critical illness framework, matching the advantage of a specialized mortality description with a contract strictly connected with the longevity phenomenon, thanks to which health insurance products are going to be increasingly issued. In particular, in the paper we propose a new contractual model in which an accelerated critical illness is offered with a special accelerated benefit in case of death for a specific cause. Referring to specific cause of death involves smaller expenses and allows for covering the death risk too. The approach combining disaggregated death probabilities and contracts growing in longevity improvement conditions, opens in our opinion new horizons in the actuarial models. Advantages for both the counterparties can be got and marketing competitiveness can be improved.

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