A macroeconomic model of consumption and investment spending: An econometric application for the economy of Cyprus.

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Abstract—A simple macroeconomic model of consumption and investment spending is specified and estimated, using time series data from the economy of Cyprus. The parameter estimates are found accurate and with plausible values. The model's dynamic behaviour seems realistic and its forecasting ability satisfactory.

Keywords—Dynamic forecasts, full information maximum likelihood method, macroeconomic models, simultaneous equation systems, single period forecasts..

I. Introduction

THe main aim of this work is the development of a simple econometric model that can be employed for the study of aggregate consumption and aggregate investment decisions for the economy of Cyprus.

The bibliography is full of examples of economic studies where both aggregate consumption and aggregate investment behaviour are examined together. A theoretical investigation of consumption - investment decisions is often found in small economic models that are concerned with trade-cycle theory ([13], [11], [12], [8]) or in models that relate to macroeconomic growth ([14], [2]). An empirical analysis of such aggregates can be found in econometric models that focus on specific sectors in the economy or relate to the whole macroeconomic system. In most cases, such frameworks of analysis are relatively large with respect to the number of behavioural equations (and endogenous variables) which they include ([9], [10], [6], [3], [7]).

The econometric model developed in this paper assumes that the main determinants of aggregate income are aggregate consumption and investment spending. These variables, expressed in their first differences, are combined in a small macroeconomic system whose parameters are estimated by means of the Full Information Maximum Likelihood (FIML) method, using annual time series data over the period 1960-1996. The model is also employed for the production of two different types of forecasts.

The proposed model is the first econometric work for the macro-economy in Cyprus in which consumption and investment are analysed together. It can be regarded as an extension of an earlier model proposed by [4] where aggregate consumption spending and national income are studied simultaneously in logarithmic deviation form. Comparisons between the estimates and between the forecasts from the two cases will be provided in the sections that follow.

The plan of this paper is the following: In section 2 the model is specified. In section 3 a brief description of the econometric method is provided. Also in that section the reduced form of the model is derived. Section 4 relates to the data and section 5 presents the estimates. Section 6 summarises the forecasts and section 7 gives the conclusion.

II. The Model

Let us suppose that aggregate consumption and investment plans in the economy are determined by the simultaneous equation system,

$$\begin{aligned} \Delta C_t &= a1\,.\Delta Y_t + a2\,.\Delta Y_{t-1} \\ \Delta I_t &= b1\,.\Delta C_t + b2\,.\Delta C_{t-1} + b3\,.\Delta I_{t-1} \\ \Delta Y_t &= c1\,.\Delta Y_{t-1} + c2\,.\Delta Y_{t-2} \end{aligned} \tag{1}$$

where C_t is planned aggregate real consumption expenditure over period t,
I_t is planned aggregate real investment expenditure over period t,
Y_t is aggregate real income over period t,
$a1,\ a2,\ b1,\ b2,\ b3,\ c1$ and $c2$ are constant parameters,
$\Delta(\ )$ denotes the difference operator and
$t = 1,\ 2,\ 3\ \ldots\ T$, denotes the time period.

In the above system the variables ΔC_t, ΔI_t and ΔY_t will be assumed to be endogenous while all the lagged variables will be assumed to be predetermined. In this way the model is properly identified and can be estimated by an appropriate econometric technique.

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Let us now consider the equations in a little more detail. The first equation is a consumption function showing that changes in current consumption depend on changes in current income and on changes in income one-period earlier. The second equation is an investment function indicating that changes in investment this period are affected by current and one-period lagged changes in consumption and also by one-period lagged changes in investment[1]. The last equation shows that changes in the level of income in any period could be related to past changes one period and two periods earlier.

If we ignore that the variables are in first difference, we observe that the first two equations of the model are similar, but dynamically more flexible, to those used in the common trade-cycle models. [13] and [8], for example, use a simple Keynesian consumption function with a Robertsonian lag. In the present case the function is extended to include also the current income variable. The investment function is also similar to that used by Samuelson, but with the assumption that $b1$ and $b2$ are not necessarily equal. In addition, the investment function in this model allows for a possible direct influence of last period's investment on current investment. Finally, the income identity constraint (which is commonly employed in trade-cycle and long run growth models) is replaced by the income autoregressive in order to allow for a more realistic representation for the movements in the available data set.

III. Estimation Method and Reduced Form

The parameters of the linear system (1) can be estimated, using the Full Information Maximum Likelihood (FIML) method. This method has more desirable asymptotic properties, compared to other system techniques[2]. Rearranging the system in stochastic form, we get,

$$\mathbf{YA} = \mathbf{XB} + \mathbf{U} \tag{2}$$

where $\mathbf{Y}$ is a $\mathrm{T} \times \mathrm{N}$ matrix of observable current endogenous random variables,
$\mathbf{X}$ is a $\mathrm{T} \times \mathrm{K}$ matrix of predetermined variables,
$\mathbf{U}$ is a $\mathrm{T} \times \mathrm{N}$ matrix of disturbances,
$\mathbf{A}$ and $\mathbf{B}$ are $\mathrm{N} \times \mathrm{N}$ and $\mathrm{K} \times \mathrm{N}$ matrices of unknown parameters that we wish to estimate,
T is the number of observations available,
N is the number of endogenous variables and
K is the number of predetermined variables.

It is important to assume that the vectors corresponding to the columns of $\mathbf{U}$, follow a multivariate normal distribution and for each value of t the matrix has mean zero and an unknown variance $\mathbf{\Sigma}$. In addition, the elements in each column vector are assumed serially independent.

[1] Over the relevant sample period, the Central Bank of Cyprus adopted a fixed interest rate policy. Thus the effect of interest rate variations on investment can be ignored in this paper.

[2] See [1], ch. 7.

Under the above conditions, the logarithmic likelihood function, $\ln \mathrm{L}$, can be expressed as

$$\begin{aligned} \ln \mathrm{L} = &- (\mathrm{NT}/2).\ln(2\pi) + \mathrm{T}.\ln\|\mathbf{A}\| - (\mathrm{T}/2).\ln|\mathbf{\Sigma}| \\ &- (1/2)\mathrm{tr}[\Sigma^{-1}(\mathbf{YA}-\mathbf{XB})''(\mathbf{YA}-\mathbf{XB})] \end{aligned} \tag{3}$$

where $\|\mathbf{A}\|$ is the absolute value of the determinant of $\mathbf{A}$ and tr[ ] denotes the trace.
Setting ($d \ln \mathrm{L} / d \Sigma$) equal to zero, it is possible to derive the expression

$$\mathbf{\Sigma} = \mathrm{T}^{-1}(\mathbf{YA} - \mathbf{XB})'(\mathbf{YA} - \mathbf{XB}) \tag{4}$$

Substitution in (3), gives the concentrated log-likelihood, $\ln \mathrm{L}^*$, given as

$$\begin{aligned} \ln \mathrm{L}^* = &- (\mathrm{T}/2).\{\ \ln\left|\mathrm{T}^{-1}(\mathbf{Y} - \mathbf{XBA}^{-1})''.(\mathbf{Y} - \mathbf{XBA}^{-1})\right|\ \} \\ &+ \text{constant} \end{aligned} \tag{5}$$

Maximising the last function with respect to $\mathbf{A}$ and $\mathbf{B}$, gives the FIML estimates $\mathbf{A}^*$ and $\mathbf{B}^*$. Substituting these into (4), gives $\mathbf{\Sigma}^*$, the FIML estimate of the variance.

Also form (2) can be simplified as

$$\mathbf{y(t)'A = x(t)'B + u(t)'} \tag{6}$$

where $\mathbf{y(t)}$, $\mathbf{x(t)}$, $\mathbf{u(t)}$ are of order N×1, K×1 and N×1 respectively and have elements vectors that correspond to the columns of $\mathbf{Y}$, $\mathbf{X}$ and $\mathbf{U}$.

Since the higher lag order in the predetermined variables is 2, the system can be transformed to

$$\mathbf{A'y(t) - B_a\, y(t\text{-}1) - B_b\, y(t\text{-}2) = u(t)} \tag{7}$$

where $\mathbf{B_a}$ and $\mathbf{B_b}$ are both square matrices of order N×N. We can attain equality in the order of these two matrices, by adding zeros to the elements that correspond to missing lagged endogenous.

Moreover, equation (7) could be expressed as,

$$\mathbf{B_0\, y(t) + B_1\, y(t\text{-}1) + B_2\, y(t\text{-}2) = u(t)} \tag{8}$$

where $\mathbf{B_0 = A'}$, $\mathbf{B_1 = -B_a}$ and $\mathbf{B_2 = -B_b}$.

Therefore the system has a reduced form,

$$\mathbf{y(t) = \Pi_1\, y(t\text{-}1) + \Pi_2\, y(t\text{-}2) + v(t)} \tag{9}$$

where $\mathbf{\Pi_1 = (A^{-1})'B_a}$, $\mathbf{\Pi_2 = (A^{-1})'B_b}$ and $\mathbf{v(t) = (A^{-1})'u(t)}$.

Also, its characteristic equation can be expressed as

$$| \mathbf{B_0} \lambda^2 + \mathbf{B_1} \lambda + \mathbf{B_2} | = \mathbf{0}. \qquad (10)$$

IV. THE DATA

The parameters of the model are estimated by means of annual time series data from the economy in Cyprus. The data, obtained from government statistical publications, relates to the period 1960-2000. Estimation is carried out over the sample period 1960-1996 while four observations for the period 1997-2000 are preserved for each variable for forecasting purposes[3].

Data for the aggregate consumption variable, C_t, is obtained from the available published time series at constant 1980 prices[4].

Data for investment, I_t, is obtained from the available gross capital formation series at constant 1980 prices.

In the case of aggregate income, Y_t, data is obtained from the available series of GDP, also at constant 1980 prices.

Both the consumption and income series are expressed in terms of per capita values, using the published series of annual population figures.

Finally, for estimation, the three data series employed in the model are transformed to their first difference.

V. THE ESTIMATES

The FIML estimates of the model for the period 1960-1996, are presented in table I, below. We notice that all the estimates are significant at a level less than 10 percent.

The value of $a1$ is highly significant and shows that a unit rise in ΔY_t causes a rise in ΔC_t by 0.5441 units. The estimate in this model is higher than the marginal propensity estimate found in [4]. In that case the short-run MPC was found to lie in the range 0.40 to 0.47.

A positive value is also obtained for $a2$, the sensitivity of changes in consumption with respect to changes in income a period earlier. More specifically, a *ceteris paribus* rise in ΔY_{t-1} by one unit, makes ΔC_t to rise by 0.1475 units.

The values of $b1$ and $b2$ are also significant and both positive, showing a direct relation between changes in investment and changes in consumption either in the current or the previous period. More specifically, a unit change in ΔC_t causes ΔI_t to change by around 151 units, while a unit change in ΔC_{t-1} causes ΔI_t to change by around 209 units. The value of $b3$ is negative, showing that a unit rise in ΔI_{t-1} is followed by a fall in ΔI_t by about 0.44 units.

Finally, we observe that ΔY_t can also be related to its lagged values. A unit changes in ΔY_{t-1} causes a change in ΔY_t by 0.2479 units while a unit change in ΔY_{t-2} causes a change in ΔY_t by 0.3618 units.

Using the characteristic equation of the system, we derive the relevant latent roots which are shown in the last row in table 1. The first root, λ_1, corresponds to $b3$ and therefore relates to the investment function. The other two roots, λ_2 and λ_3, are common to the consumption and income equations. All the roots have modulus less than 1 and therefore the system is stable[5].

The estimates presented in table I, may not be so easy to use for policy making purposes since these are just measures of sensitivity for the variables. Elasticity measures on the other hand can be found more useful as they denote percentage changes. Such elasticity measures are derived using the estimated sensitivities and the structural form relations. Their values and standard errors are presented in table II.

Considering, for example, the income elasticity of consumption, $g^{c_t}_{y_t}$, we notice that this is positive and less than one. This is what it might be expected since this denotes the ratio of the marginal to the average propensity to consume. It shows that a unit percentage rise in real per capita income would cause real per capita consumption to rise by 0.8118 percent, over the same period[6].

The consumption elasticity of investment, $g^{I_t}_{c_t}$, is also positive indicating the positive relation between the two variables. A unit percentage rise in real per capita consumption would account for a 0.5798 percent rise in real investment over the same period.

[3] In [4] the model is estimated over the period 1960-1995 and excludes the 1975 observation. Thus the sample size for this project is larger only by two data observations. Using almost the same sample size, could allow us to make more accurate comparisons on the estimates and forecasts of the two models.

[4] All the data series can be collected from the following statistical booklets, published by the
Statistical Service of the Ministry of Finance in Cyprus:
-*Economic Report 1995 & 1996,*
-*Historical Data on the Economy of Cyprus 1960 -1991,*
-*National Accounts 1999,*
-*National Accounts 2000.*

[5] The value of R-sqr provides a measure of fit of the model and is statistically analogous to the square correlation coefficient of single equation regressions. In addition, a vector autocorrelation test of the first order conducted, showed no evidence of significant serial correlation. This is verified by the small value of the LR statistics.

[6] Notice that the values in table 2 refer to percentage changes of variables in levels whereas those in table 1 refer to unit changes of variables in differences.

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Table I. Structural form estimates

parameter	estimate	st. error	t-value	significance
$a1$	0.5441	0.0840	6.4731	0.000
$a2$	0.1475	0.0654	2.2564	0.030
$b1$	150.9100	80.5355	1.8738	0.069
$b2$	208.9730	89.5764	2.3329	0.025
$b3$	-0.4443	0.1293	-3.4353	0.002
$c1$	0.2479	0.1165	2.1287	0.040
$c2$	0.3618	0.1293	2.7980	0.008
max lnL = 95.0569, T = 34, λ_1 = -0.4443, λ_2 = 0.7381, λ_3 = -0.4901, $LR(5)$ = 1.2258, R-sqr = 0.4987				

Table II. Derived consumption, investment and income elasticities

elasticity	value	st. error	elasticity	value	st. error
$g^{c\,t}_{y\,t}$	0.8118	0.1253	$g^{I\,t}_{c\,t-2}$	-0.6158	0.2640
$g^{c\,t}_{y\,t-1}$	-0.5434	0.1404	$g^{I\,t}_{I\,t-1}$	0.4869	0.1133
$g^{c\,t}_{y\,t-2}$	-0.1890	0.0838	$g^{I\,t}_{I\,t-2}$	0.2523	0.0734
$g^{c\,t}_{c\,t-1}$	0.9525	0.0000	$g^{y\,t}_{y\,t-1}$	1.1460	0.1070
$g^{I\,t}_{c\,t}$	0.5798	0.3094	$g^{y\,t}_{y\,t-2}$	0.0978	0.7213*
$g^{I\,t}_{c\,t-1}$	0.2022	0.4596*	$g^{y\,t}_{y\,t-3}$	-0.2686	0.0960

***** Not significant

We notice that all the elasticity measures presented are significant (at a level less than 10 percent), apart from those for $g^{I\,t}_{c\,t-1}$ and for $g^{y\,t}_{y\,t-2}$.

Next, let us consider the long-run parameters of the model. The analysis will be restricted only to changes that arise from an initial change in income. In this model, such a change will not stop at the first period, but it will continue to have an influence on the system in subsequent periods too. An analogous treatment with this particular case can be found in [5].

In the cases of initial changes in consumption or investment, the long-run parameters can be determined in a way very similar to that which relates to a change in a variable of the system which is strictly exogenous.

The estimates and the standard errors are presented in table III.

The first parameter, $e_{c^*\,y}$, shows that a unit change in real income, from an equilibrium level Y^*, causes the equilibrium value of real consumption, C^*, to change by 1.2279 units over the long-run. Expressed as an elasticity, a unit percentage change in Y^*, causes C^* to change by 1.83 percent over the long-run.

The second parameter, $e_{I^*\,y}$, shows that an initial change in real income by one unit, makes the equilibrium value of real investment, I^* to change by 305.9623 units. Alternatively, a unit percentage change in Y^*, causes I^* to change by 1.84 percent over the long-run.

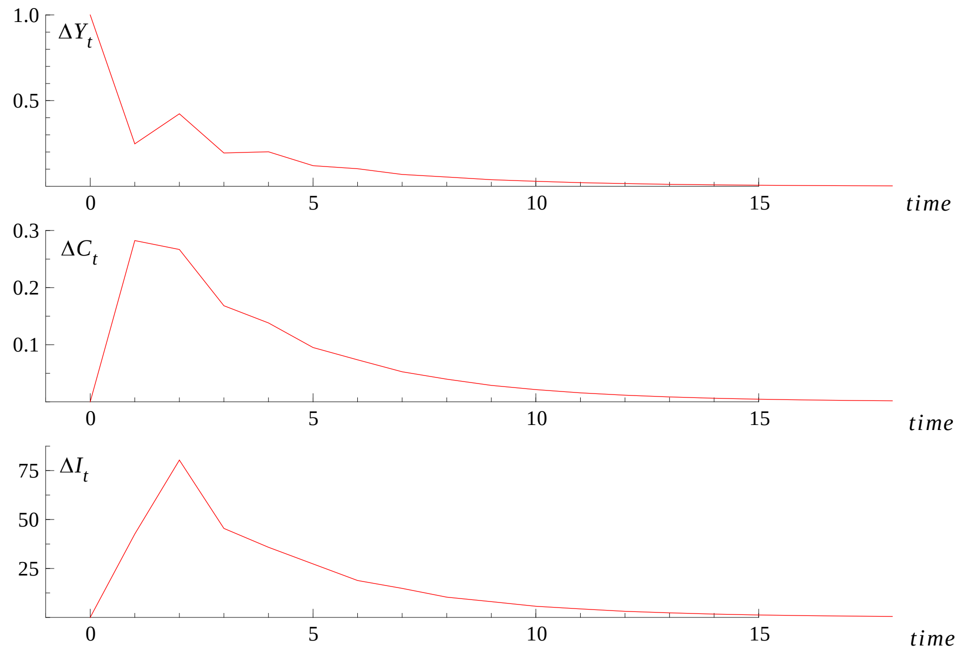
Finally, the income parameter, shows that an initial change in real income by one unit, causes the equilibrium value of

Table III. Long-run parameters

parameter	sensitivity	st error	elasticity*	st error
$e_{c^*\,y}$	1.2279	0.5263	1.8322	0.7853
$e_{I^*\,y}$	305.9623	131.1510	1.8416	0.7894
$e_{y^*\,y}$	2.5622	1.0983	2.5622	1.0983

***** The elasticities are normalised on nearest value to sample means

Figure 1. Time plots of endogenous variables.



that variable to change by 2.5622 units over the long-run. Expressed as an elasticity, it shows that an initial unit percentage rise, makes that variable to change from its equilibrium by 2.56 percent. In [4], the logarithmic deviation model gives an elasticity value of about 2.88 percent.

It is also possible to demonstrate graphically how the time paths of the variables, change through time. For that purpose, at a starting period zero, ΔY_t can be assigned a value equal to 1 unit. That initial change will have an effect on the values of the endogenous variables (including ΔY_t) one period later, then a new effect two periods later and so on.

The plots, shown in figure 1, indicate that the change in ΔY_t continues through time but at smaller values, until it converges to zero. Changes in ΔC_t and also in ΔI_t, build up immediately after the initial distortion, reach a maximum and then start falling and eventually diminish.

The maximum for ΔC_t occurs with a delay of one period while that for ΔI_t occurs after a delay of two periods.

The results of course, are in agreement with the postulates of economic theory that require both consumption and investment spending to intensify after a significant upturn in economic activity.

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VI. Forecasting

The model is also employed for the production of forecasts over the post-sample period 1997-2000. The forecasted values are then compared with the actual observations that have been preserved and the forecasting errors are derived. The results are presented in tables IV and V. So as to enable comparisons with the log-deviation model, both the forecasts and the actual values are expressed in logarithms.

Table IV. Single period forecasts for the period 1997-2000.

variable	year	actual value	forecast	error
$\ln c_t$				
	1997	0.6584	0.6486	-0.0098
	1998	0.7369	0.6664	-0.0705
	1999	0.7461	0.7555	0.0094
	2000	0.7941	0.7692	-0.0249
rmse = 0.0380				
$\ln I_t$				
	1997	6.1070	6.2192	0.1121
	1998	6.2162	6.1748	-0.0414
	1999	6.2096	6.2474	0.0378
	2000	6.2849	6.2352	-0.0496
rmse = 0.0674				
$\ln y_t$				
	1997	1.0097	1.0137	0.0040
	1998	1.0500	1.0165	-0.0336
	1999	1.0831	1.0644	-0.0187
	2000	1.1220	1.1048	-0.0172
rmse = 0.0211				

Table V. Dynamic forecasts for the period 1997-2000.

variable	year	actual value	forecast	error
$\ln c_t$				
	1997	0.6584	0.6486	-0.0098
	1998	0.7369	0.6583	-0.0786
	1999	0.7461	0.6663	-0.0798
	2000	0.7941	0.6717	-0.1179
rmse = 0.0815				
$\ln I_t$				
	1997	6.1070	6.2192	0.1121
	1998	6.2162	6.2277	0.0115
	1999	6.2096	6.2362	0.0266
	2000	6.2849	6.2421	-0.0427
rmse=0.0617				
$\ln y_t$				
	1997	1.0097	1.0137	0.0040
	1998	1.0500	1.0215	-0.0286
	1999	1.0831	1.0296	-0.0535
	2000	1.1220	1.0343	-0.0877
rmse=0.0533				

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Table VI. Root mean square errors of the forecasts

variable	single-period forecasts*	dynamic forecasts	trend forecasts
$\ln c_t$	0.0380	0.0815	0.1664
$\ln I_t$	0.0674	0.0617	0.1401
$\ln y_t$	0.0211	0.0533	0.1739

* Figures in parenthesis indicate the rmse values of the log-deviation model

Single period forecasts, listed in table IV, give the forecasted value for a variable in a specific period in the post-sample, by incorporating each time actual values of the variables from the earlier period. We observe that the errors are quite low for all the three variables. In absolute terms, the error for consumption ranges from around 0.01 to 0.07 while for investment it ranges from around 0.04 to 0.11. In the case of income the error is even lower ranging from around 0.004 to 0.034.

Dynamic forecasts are listed in table V. In this category, the forecasting procedure is initiated with some known past values and generates forecasts of the variables for a number of periods ahead. In absolute terms, the errors fall approximately in the ranges 0.01 to 0.12. for consumption, 0.01 to 0.11 for investment and 0.004 to 0.09 for income.

A better assessment of the model's predictive power can be made from the use of the root mean square errors (rmse) of the generated forecasts. These are listed in table VI together with the rmse that are obtained from naïve trend projections and relate to the same post-sample period. The values in the brackets below the consumption and income variables correspond to the rmse of the forecasts of the log-deviation model[7].

Considering first single period forecasts, we notice that the lower error of 0.0211 corresponds to the income forecasts, then comes the error of 0.0380 for consumption forecasts and last follows the error of 0.0674 for investment forecasts.

In the case of dynamic forecasts, income has again the lowest error value of 0.0533, then comes investment with 0.0617 and finally consumption with 0.0815. Therefore, single period income and consumption forecasts are superior to the corresponding dynamic forecasts, but single period

investment forecasts are slightly inferior to the relevant dynamic forecasts.

Both single period and dynamic forecasts are found superior to naïve trend forecasts for all the variables. In addition, the forecasts of the proposed model appear superior to those derived from the log-deviation model. In three out of the four comparisons the current model has lower errors. The exception is the error that relates to the single period forecast of consumption which is found marginally higher than that which corresponds to the log-deviation case.

VII. CONCLUSION

A simple model of the macro-economy of Cyprus was specified and estimated. The estimates were found highly significant and in agreement with the main principles of economic theory. Also some dynamic properties of the model were analysed and were found realistic.

An additional test of the validity of this work is the accuracy of the derived forecasts. In all the cases these were found to outperform the relevant naïve trend forecasts. Also in most of the comparisons these forecasts were found superior to those that have been derived by the log-deviation model for the consumption - income relations.

The results suggest that the model could be employed for economic analysis and also it could be considered as a good basis for further research.

[7] The post-sample period for the log-deviation model ranges from 1996 to 2000. Thus, it includes an additional forecast value, that of the year 1996.

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