Digital Smith Predictor for Control of Unstable and Integrating Time-delay Processes

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Abstract—Time-delays (dead time) are found in many processes in industrial practice. Time-delays are mainly caused by the time required to transport mass, energy or information, but they can also be caused by processing time or accumulation. This paper deals with a design of algorithms for digital control of the unstable and integrating time-delay processes using one suitable modification of the Smith Predictor (SP). This digital modification of the Smith Predictor is based on Linear Quadratic (LQ) method. A minimization of the quadratic criterion is realized using spectral factorization. The designed algorithms have universal usage; they are suitable for control of stable, non-minimum phase, unstable and integrative time-delay processes. The main contribution of this paper is design and simulation verification of this Smith Predictor for control of the unstable and integrative processes, because classical continuous Smith Predictors are not suitable for control of such processes. Of course, some continuous-time modifications have been designed for control of like these processes. The designed algorithms for control of individual processes influenced by external disturbance were verified. The program system MATLAB/SIMULINK was used for simulation of designed algorithms.

Keywords—Digital control, Integrating process, LQ control, Polynomial approach, Simulation of control loops, Smith predictor, Time-delay, Unstable process.

1. INTRODUCTION

Time-delay appear in many processes in industry and other fields, including economical and biological systems. They are caused by some of the following phenomena [1]:

- the time needed to transport mass, energy or information,
- the accumulation of time lags in a great numbers of low order systems connected in series,
- the required processing time for sensors, such as analyzers; controllers that need some time to implement a complicated control algorithms or process.

Time-delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of controllability, observability, robustness, optimization, adaptive control, pole placement and particularly stability and robust stabilization for this class of systems, has been one of the main interests for many scientists and researchers during the last five decades.

A part of time-delay systems can be unstable or have integrating properties. Typical examples of such processes are e.g. pumps, liquid storing tanks, distillation columns or some types of chemical reactors.

Most authors are designing continuous-time algorithms for control of such processes. Integrating and unstable processes with a time-delay often cannot be controlled by usual controllers designed without consideration of the dead-time. There are various ways to control such systems. Several tuning rules for PI or PID controllers in the classical feedback closed-loop continuous-time structure have been presented in literature for these systems, see e.g. [2] – [7]. But when processes include long time-delay, the performances of these classical controllers become worsen [8]. In these cases, the use of a time-delay compensator in the structure of the closed-loop control system can be available [9].

The first time-delay compensation algorithm was proposed by Smith [10] in 1957. This time-delay compensator (TDC) known as the Smith predictor (SP) contained a dynamic model of the time-delay process and it can be considered as the first model predictive algorithm. Control results of a good quality can be achieved by modified Smith predictor methods, see e.g. in [11] – [17]. The control scheme 2DOF (Two Degrees Of Freedom) is used in [18] - [20]. The design of controllers using polynomial approach [21], [22] can be found in [23] and the control system structure with two feedback controllers is proposed in [24]. The idea of the IMC (Internal Model Control) is employed in [25].

The problems of continuous-time control of integrating or unstable time-delay systems including the robustness, disturbance rejection and the extension of suitable compensators have been analyzed in other articles, see e.g. [26] - [34].

Historically first modifications of time-delay algorithms were proposed for continuous-time (analog) controllers. In industrial practice the implementation of the time-delay compensators on analog technique was difficult. Therefore the
Smith Predictors and its modified versions can be implemented since 1980s together with the use of microprocessors in the industrial controllers. In spite of the fact that all these algorithms are implemented in digital platforms, most of the literature analyzes, as mentioned above, only the continuous-time version.

The first digital time-delay compensators are presented e.g. in [35] – [38]. Some Self-tuning Controller (STC) modifications of the digital Smith Predictors (STCSP) are designed in [39] – [41]. Two versions of the STCSP were implemented into MATLAB/SIMULINK Toolbox [42], [43]. The scope of paper [44] is a design and an analysis of 2DOF discrete time-delay compensators for stable and integrating processes, the simple robust discrete time-delay compensator for unstable processes is proposed in [45].

It is well known that classical analog Smith Predictor is not suitable for control of unstable and integrating time-delay processes. The designed digital LQ Smith Predictor eliminates this drawback.

The paper is organized in the following way. The general problem of a control of the time-delay systems is described in Section 1. The principle of the continuous-time Smith Predictor is introduced in Section 2 and digital version in Section 3. Three modifications of digital controllers that are used for self-tuning versions SPs are proposed in Section 4. Section 5 contains brief description of the recursive identification procedure. Simulation configuration is presented in Section 6. Results of simulation experiments are summed in Section 7.

II. PROCESS MODELS

Consider a continuous-time dynamical linear SISO (single input $u(t)$ – single output $y(t)$) system with time-delay $L$.

$$G_e(s) = G(s)e^{-LT}$$

(1)

where $G(s)$ is an unstable or an integrating time-delay free part of the process and the transfer function of a pure transportation lag is $e^{-LT}$, where $s$ is complex variable. A more complete description of the process must include external disturbances, which are normally represented in the linear model as an additive signal at process output.

This paper presents digital control of the unstable second order systems with time-delay which can be described by the following continuous-time transfer function:

1) System with one unstable pole:

$$G_1(s) = \frac{K}{(Ts+1)(Ts-1)}e^{-LT}$$

(2)

2) System with two unstable poles:

$$G_2(s) = \frac{K}{(Ts-1)(Ts-1)}e^{-LT}$$

(3)

3) Oscillatory unstable system:

$$G_3(s) = \frac{K}{Ts^2 + 2\zeta Tse^{-LT}}; \quad 0 < \zeta < 1$$

(4)

4) Non-minimum phase unstable system with one unstable pole:

$$G_4(s) = \frac{K(Ts-1)}{(Ts+1)(Ts-1)}e^{-LT}$$

(5)

5) Non-minimum phase unstable system with two unstable poles:

$$G_5(s) = \frac{K(Ts-1)}{(Ts-1)(Ts-1)}e^{-LT}$$

(6)

6) Oscillatory non-minimum phase unstable system:

$$G_6(s) = \frac{K(Ts-1)}{Ts^2 + 2\zeta Tse^{-LT}}$$

(7)

where $K$ is static gain, $T$, $T_1$, $T_2$, $T_3$ are time constants and $\zeta$ is damping factor.

The following integration systems were chosen for verification of the proposed digital SP algorithm:

7) Integrating system with one stable pole:

$$G_7(s) = \frac{K}{s(Ts+1)}e^{-LT}$$

(8)

8) Integrating system with one unstable pole:

$$G_8(s) = \frac{K}{s(Ts-1)}e^{-LT}$$

(9)

9) Double integrating system:

$$G_9(s) = \frac{K}{s^2}e^{-LT}$$

(10)

III. DIGITAL SMITH PREDICTOR

The discrete versions of the SP and its modifications are more suitable for time-delay compensation in industrial practice. The block diagram of a digital SP (see [39], [40]) is shown in Fig. 1. The function of the digital version is similar to the classical analog version.

![Fig. 1 Block diagram of a digital Smith Predictor](image)

Number of higher order industrial processes can be approximated by a reduced order model with a pure time-delay. In this paper the following second-order linear model with a time-delay is considered.
The term \( z^{-d} \) represents the pure discrete time-delay. The time-delay is equal to \( DT_0 \), where \( T_0 \) is the sampling period.

The block \( G_d(z^{-1}) \) represents process dynamics without the time-delay and is used to compute an open-loop prediction. The numerator in transfer function (11) is replaced by its static time-delay and is used to compute an open-loop prediction.

For the second order model (11) first compensator has the form

\[
G_n(z^{-1}) = \frac{(b_1 + b_2)z^{-1}}{1 + a_1z^{-1} + a_2z^{-2}}; \quad b_2 = b_1 + b_2
\]  

(12)

and second compensator is given by the transfer function

\[
G_q(z^{-1}) = \frac{b_1z^{-1} + b_2z^{-2}}{b_2z^{-1}}z^{-d}
\]  

(13)

A. Design of Primary Polynomial 2DOF Controller

\[
\begin{align*}
\begin{bmatrix}
& b_r & 0 & 0 & 1 \\
& 0 & b_r & 0 & a_1^{-1} \\
& 0 & 0 & b_r & a_2^{-1} \\
& 0 & 0 & 0 & -a_2
\end{bmatrix}
\begin{bmatrix}
 & q_0 \\
 & q_1 \\
 & q_2 \\
 & q_3
\end{bmatrix}
= \begin{bmatrix}
 & d_0 + 1 - a_1 \\
 & d_2 + a_1 - a_2 \\
 & d_3 + a_2 \\
 & d_4
\end{bmatrix}
\end{align*}
\]  

(19)

An asymptotic tracking is provided by a feedforward part of the controller given by a solution of the polynomial Diophantine equation (17). A feedback part of the controller to control a second-order system with time-delay will be derived from equation (17). A system of linear equations can be obtained using the uncertain coefficients method

\[
R = r_0 = \frac{D(1)}{B(1)} = 1 + d_1 + d_2 + d_3 + d_4
\]  

(21)

The 2DOF controller output is given by

\[
a(k) = r_0 w(k) - q_0 y(k) - q_1 y(k-1) - q_2 y(k-2) + (1 - p_i) u(k-1) + p_i u(k-2)
\]  

(22)

B. Minimization of LQ Criterion

The linear quadratic control methods try to minimize the quadratic criterion by penalization the controller output

\[
J = \sum_{k=0}^{\infty} \left[ (w(k) - y(k))^2 + q_n |u(k)|^2 \right]
\]  

(23)
where \( q_e \) is the so-called penalization constant, which gives the rate of the controller output on the value of the criterion (where the constant at the first element of the criterion is considered equal to one). In this paper, criterion minimization will be realized through the spectral factorization for an input-output description of the system.

For the coefficients of the second order characteristic polynomial

\[
D_2(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2}
\]

of the closed loop the following expressions were derived [48]

\[
d_i = -\frac{m_i}{\delta} + \frac{m_2}{\delta}; \quad d_2 = \frac{m_2}{\delta}
\]

(25)

The parameters \( m_1, m_2 \) and \( \delta \) are computed as follows:

\[
\delta = \frac{\gamma + \sqrt{\gamma^2 - 4m_2^2}}{2}; \quad \gamma = \frac{m_1 - m_2}{2} + \sqrt{\left(\frac{m_1 - m_2}{2}\right)^2 - m_1^2}
\]

\[
m_0 = q_e (1 + a_1^2 + a_2^2) + b_i^2; \quad m_1 = q_e (a_1 + a_2); \quad m_2 = q_e a_2
\]

(26)

IV. PRIMARY LQ CONTROLLER OF DIGITAL SP

From the previous paragraph, it is obvious that using analytical spectral factorization, only two parameters \( d_1 \) and \( d_2 \) of the second degree polynomial \( D_2(z^{-1}) \) can be computed. This approach is applicable only for control of processes without time-delay (out of Smith Predictor). The primary controller in the digital Smith Predictor structure requires usage of the fourth degree polynomial \( D_4(z^{-1}) \) (18) in equations (17) and (20). The polynomial \( D_4(z^{-1}) \) has two different real poles \( \alpha, \beta \) or one complex conjugated pole \( z_{1,2} = \alpha \pm j\beta \) (in the case of oscillatory systems). These poles must be included into polynomial \( D_4(z^{-1}) \) (18). A suitable pole assignment was designed for both types of the processes:

1st possibility:

Polynomial (18) has two different real poles \( \alpha, \beta \) (computed from (24)) and user-defined real poles \( \gamma, \delta \). Then it is possible to write polynomial (18) as a product root of factor

\[
D_4(z) = (z - \alpha)(z - \beta)(z - \gamma)(z - \delta)
\]

(27)

and its individual parameters can be expressed as

\[
d_1 = -(\alpha + \gamma + \delta)
\]

\[
d_2 = \alpha \beta + \gamma \delta + (\alpha + \beta)(\gamma + \delta)
\]

\[
d_3 = -[(\alpha + \beta)\gamma \delta + (\gamma + \delta)\alpha \beta]
\]

\[
d_4 = \alpha \beta \gamma \delta
\]

(28)

2nd possibility:

Polynomial (18) has the complex conjugate pole \( z_{1,2} = \alpha \pm j\beta \) (computed from (24)) and user-defined real poles \( \gamma, \delta \). Then polynomial (18) has the form

\[
D_4(z) = (z - \alpha - j\beta)(z - \alpha + j\beta)(z - \gamma)(z - \delta)
\]

(29)

and its individual parameters can be expressed as

\[
d_1 = -(2\alpha + \gamma + \delta)
\]

\[
d_2 = 2\alpha \beta + \gamma \delta + (\alpha + \beta)(\gamma + \delta)
\]

\[
d_3 = \alpha \beta \gamma \delta
\]

(30)

The control algorithm based on the LQ control method contains the following steps:

- The parameters of the polynomial \( D_4(z^{-1}) \) are computed according to equations (25) and (26).
- If the polynomial (24) has the real poles \( \alpha, \beta \), its parameters are computed according to equations (28), otherwise, they are computed according to equations (30).
- The controller parameters are computed using matrix equation (19) and equation (21).
- The controller output is given by equation (22).
- Penalization of the controller output is performed by setting \( q_e \geq 0 \).

With increased penalization constant, the amplitude of the controller output decreases and thereby, the flow of the process output is smoothed and any possible oscillations or instability are damped.

V. SIMULATION VERIFICATION AND RESULTS

Simulation is useful tool for the synthesis of control systems, allowing us not only to create mathematical models of a process but also to design virtual controllers in a computer. The mathematical models provided are sufficiently close to a real object that simulation can be used to verify the dynamic characteristics of control loops when the structure or parameters of the controller change. The models of the processes may also be excited by various random noise generators which can simulate the stochastic characteristics of the processes noise signals with similar properties to disturbance signals measured in the machinery. A simulation verification of the designed predictive algorithm was performed in MATLAB/SIMULINK environment. It is possible to influence the output of the process with the non-measurable disturbance \( d \). The designed digital Smith Predictor has universal usage for control of a large group of processes with time-delay.

Because the range of this paper is limited, only the following models with an exacting dynamic behavior were used for simulation experiments:

System with two unstable poles – (3)

\[
G_s(s) = \frac{2}{(5s - 1)(2s - 1)} e^{-5t}
\]

(31)

Oscillatory unstable non-minimum phase system – (7)

\[
G_s(s) = \frac{2(4s - 1)}{4s^2 + 2s - 1} e^{-4s}
\]

(32)

Integrating system with one unstable pole – (9)

\[
G_s(s) = \frac{2}{s(5s - 1)} e^{-5s}
\]

(33)
Let us now discretize (31), (32) and (33) using a sampling period $T_s = 2\, \text{s}$. The discrete forms of these transfer functions are (see (11))

$$G_2(z^{-1}) = \frac{0.6516 z^{-1} + 1.0386 z^{-2}}{1 - 4.2101 z^{-1} + 4.0552 z^{-2}}$$

$$G_4(z^{-1}) = \frac{2.1695 z^{-1} - 3.5409 z^{-2}}{1 - 2.0653 z^{-1} + 0.3679 z^{-2}}$$

$$G_6(z^{-1}) = \frac{0.9782 z^{-1} + 1.0491 z^{-2}}{1 - 2.4918 z^{-1} + 1.4918 z^{-2}}$$

(34) (35) (36)

The processes which are described by the above mentioned transfer functions were used in the Simulink control scheme for the verification of the dynamical behavior of the individual closed control loops. In time $500 \, \text{–} \, 800\, \text{s}$ an exponential external disturbance

$$d(t) = 0.5 \left(1 - e^{-αt}\right)$$

(37)

acted on the system output. The computed poles $α$, $β$ and user-defined real poles $γ$, $δ$ are introduced for individual simulation experiments including characteristic polynomial (18). For all experiments, the penalization factor was chosen $q_0 = 1$.

A. Simulation Control of Model $G_2(z^{-1})$

The poles: $α, β = 0.3912 \pm 0.1488i; \quad γ = 0.1; \quad δ = 0.5$

The characteristic polynomial:

$$D_2(z) = z^4 - 1.3824z^3 + 0.6947z^2 - 0.1442z + 0.0088$$

The courses of the control variables are shown in Fig. 3, the quality of control is very good.

B. Simulation Control of Model $G_4(z^{-1})$

The poles: $α, β = 0.2188 \pm 0.11137i; \quad γ = 0.1; \quad δ = 0.75$

The characteristic polynomial:

$$D_4(z) = z^4 - 1.2875z^3 + 0.5077z^2 - 0.0845z + 0.0046$$

The courses of the control variables are shown in Fig. 4, the quality of control is very good.

C. Simulation Control of Model $G_6(z^{-1})$

The poles: $α, β = 0.2652 \pm 0.2752i; \quad γ = 0.1; \quad δ = 0.5$

The characteristic polynomial:

$$D_6(z) = z^4 - 1.1305z^3 + 0.5144z^2 - 0.1142z + 0.0073$$

The courses of the control variables are shown in Fig. 5, the quality of control is very good.

VI. CONCLUSION

The paper presents a new unified approach for design of the digital LQ Smith Predictor for control unstable and integrating systems with time-delay. The primary controller is based on minimization of the linear quadratic criterion. Minimization of the criterion is realized through spectral factorization. This controller was derived purposely by analytical way (without utilization of numerical methods) to obtain algorithms with easy implementability in industrial practice. Three models (two unstable and one integrating with unstable pole) were used for simulation verification. Main contribution of designed method is the universal applicability of this Smith
Predictor for digital control of large spectrum processes (stable, unstable, non-minimum phase, integrating) with time-delay.

VII. REFERENCES


