Linearity Error of Force Transducers arising from Nonlinear Elasticity

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Abstract—The linearity error of strain gage-based force transducers due to the nonlinear stress-strain behavior of an aluminum alloy is numerically calculated and analyzed using three regression models. It is found that a typical linearity error of 0.03% full scale can be obtained when the maximum deviation from the linear stress-strain behavior is in the range of 1%–2% of tensile strength depending on the regression model used. The difference between the average stress in a tension gage and the average stress in a compression gage does not appear to significantly affect the linearity error.

Keywords—linearity, load cells, strain gages, tensile modulus.

I. INTRODUCTION

A strain gage-based force transducer such as a load cell consists of an elastic element and strain gages installed on the surface of the elastic element and converts a load-induced deformation into an electrical signal proportional to the load. Force transducers have been used in robots for measuring forces exerted on joints or forces generated by actuators [1]. In general, the electrical output of the transducer is a nonlinear function of the input deformation, typically expressed in microstrain units [2]. The maximum deviation of the transducer output from the straight line between the transducer output with no load applied (zero output) and the output at rated capacity (full scale output) is referred to as the linearity error.

Commercial load cells are ranked into differing accuracy classes. A specific accuracy class specifies a combined error including temperature effects, linearity error and hysteresis, typically less than 0.1% of full scale output [3]. Various methods for compensating for these errors have been proposed and analyzed [4]–[7].

It was experimentally determined that microstructural changes induced from heat treatments can affect the hysteresis performance of force transducers [8]. It was also found that the linearity and hysteresis performance parameters of a overloaded load cell can be essentially restored by subjecting the load cell to conditioning similar to that which it received in initial manufacture [9]. These findings suggest that deviations from the previous state of crystalline structure resulting from a particular type of conditioning may affect the elastic response of the material, making the degree of nonlinearity more or less severe depending on the conditioning.

In this paper, the contributions of the nonlinear elastic response of an aluminum alloy to the transducer linear error will be numerically calculated and analyzed. The nonlinear stress-strain behavior will be modeled using three regression methods, and it will be shown that the nonlinear elasticity may cause the transducer linearity error.

II. LINEARITY ERROR

A bending beam load cell shown in Fig. 1 is designed to produce an output signal \( V_o/V_s \) (mV/V) given by

\[
V_o/V_s = \frac{R_2}{R_1 + R_4} - \frac{R_1}{R_1 + R_2},
\]

where the gage resistors, \( R_1 \) and \( R_3 \) (nominal resistance 350 \( \Omega \)), are subjected to an average tensile stress of about 40 MPa and \( R_2 \) and \( R_4 \) are subjected to an average compressive stress of 40 MPa when a 1 kgf load is applied as shown in the Wheatstone bridge circuit in Fig. 2. For the load cell shown in Fig. 1, Wheatstone bridge nonlinearity does not contribute to the transducer linearity error [10]. Let \( E \) be the linearity error given by

\[
E = \frac{V_o/V_s|_{\text{half load}} - V_o/V_s|_{\text{full load}}}{2}.
\]

The full scale output \( V_o/V_s|_{\text{full load}} \) is the output signal under full load (1kgf). A tensile modulus of 73 GPa results in an average strain of 550 \( \mu \epsilon \) (microstrain) under full load, which in turn yields a change in resistance of \( K \times 0.00055 \times 350 \, \Omega = 0.38 \, \Omega \), where \( K = 2 \) is the gage factor. Then from Eq. (1)

\[
V_o/V_s|_{\text{full load}} = \frac{350.38}{700} - \frac{349.62}{700} \approx 1.08 \, mV/V.
\]

Ideally the half scale output should be exactly one half (0.54 mV/V) of the full scale output. However, commercial load cells show deviations from this ideal linear behavior. It has been observed in our lab that the linearity error of the load cell shown in Fig. 1 is typically 0.03% full scale (0.0003 mV/V), or less.
III. NONLINEAR ELASTICITY

Most metals are known to exhibit linear elasticity in the elastic region as illustrated in Fig. 3. In this work, it is assumed that the aluminum alloy machined to make the load cell in Fig. 1 has a nonlinear stress-strain property as illustrated in Fig. 3 (dashed curve), and only the contributions of this material nonlinearity to the transducer linear error will be examined although other factors such as hyperelastic properties of strain gage backing materials used in the industry may also contribute to the linearity error.

To characterize the nonlinear stress-strain behavior of the aluminum alloy, three regression models are considered as shown in Fig. 4. A third degree polynomial model is shown as solid curve; a third order Ogden model conventionally used to describe the nonlinear stress-strain behavior of rubbers is shown as dotted curve [11]; and an inverted sinusoidal curve is shown as dashed curve. The maximum deviation from the linear stress-strain behavior is set to approximately 1% of the tensile strength (320 MPa) of the aluminum alloy for the three models.

IV. FINITE ELEMENT ANALYSIS

Commercially available finite element analysis software was used to compute the stress distribution over the gage areas. The 3D load cell model consists of nine parts: the body, the four gage areas (the dimension of each gage area is 3 mm × 1.5 mm), and the four transitional “wells” that surround the gage areas (except the surfaces) and properly connect the elements in two adjacent meshes as shown in Fig. 5. The element size of the body mesh was set to 1 mm. The gage areas are composed of much finer elements of 0.1 mm to provide smoother stress distributions.
V. SIMULATION RESULTS

The distribution of stress in the horizontal direction (positive going from R₁ to R₂ in Fig. 1) under full load (1 kgf) is shown in Fig. 6. Salient stress concentrations are shown over the gage areas. The maximum stress (46.9 MPa) was produced in R₁ and another tension gage R₄ exhibited 4% less peak stress (45.01 MPa); the minimum stress (−46.9 MPa) was produced in R₃ and the peak stress over R₂ was −45.01 MPa.

![Fig. 6 Stress distribution.](image)

<p>| Table I. Effects of the distance between R₁ and R₂ (sinusoidal stress-strain model). |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>d (mm)</th>
<th>avg. stress in R₁ (MPa)</th>
<th>avg. stress in R₂ (MPa)</th>
<th>avg. stress ratio [R₂/R₁]</th>
<th>Vₛ/Vₛ (half load) (mV/V)</th>
<th>Vₛ/Vₛ (full load) (mV/V)</th>
<th>Linearity error (% full scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>39.336</td>
<td>−36.465</td>
<td>0.927</td>
<td>0.5359</td>
<td>1.0711</td>
<td>0.0325</td>
</tr>
<tr>
<td>42</td>
<td>39.334</td>
<td>−38.021</td>
<td>0.967</td>
<td>0.5469</td>
<td>1.0932</td>
<td>0.0310</td>
</tr>
</tbody>
</table>

A. Distance between R₁ and R₂

Let d be the distance between R₁ and R₂ (d = 12 mm in Fig. 1). Consider another load cell with d = 42 mm shown in Fig. 7. This load cell is designed to produce comparable stresses over the gage areas. The linearity error obtained from the aforementioned regression models is shown in Fig. 8. The sinusoidal model results in the largest error due to its steepest initial slope (see Fig. 4), whereas the Ogden model results in the smallest error due to its least severe degree of nonlinearity up to 2,000 microstrain (see Fig. 4).

![Fig. 8 Linearity error when d = 12 mm and d = 42 mm.](image)

Table I shows how the ratio of the average stress in R₂ to the average stress in R₁ gets closer to unity as d gets larger when the sinusoidal model is employed (fourth column). The linearity error in the last column, however, is not significantly affected by the change in d. Thus, the difference between the average stress in the tension gage and the average stress in the compression gage does not appear to play a significant role in determining the linearity error as far as the contributions of the nonlinear elasticity are concerned.

![Fig. 9 Linearity error with varying stress-strain nonlinearity.](image)

B. Degree of Stress-Strain Nonlinearity

As mentioned above, the maximum deviation from the linear stress-strain behavior was set to about 3.2 MPa (1% of tensile strength) for the results presented thus far. Fig. 9 shows how the linearity error of the load cell in Fig. 1 changes as the degree of stress-strain nonlinearity varies. Assuming that the linearity error should not exceed 0.03% full scale, the degree of stress-strain nonlinearity should be less than 1% of tensile strength if the actual stress-strain behavior of the aluminum alloy follows the sinusoidal model, or less than 2% of tensile strength if it follows the polynomial model.

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REFERENCES


