Abstract—Autotuning principles usually combine relay feedback tests with a control synthesis. This paper is focused on the first part of this scheme, i.e. relay plant identification for continuous-time plants. The estimation of the controlled system parameters plays the key role in the quality of control. There are many types of relays used in feedback relay schemes. The contribution deals with four ones of them unbiased and biased relays without or with hysteresis.

Most of industrial plants can be satisfactory estimated by a first or second order linear stable with a time delay term. The main relay parameters are the asymmetry, hysteresis and amplitudes. The aim of this paper is to study and analyze the influence of these parameters for the quality of estimation of the gain, time constant and time delay. As a result, some recommendations for settings of relay features can be given.

All simulations were performed in Matlab and Simulink program environment. Identified plant parameters then can be utilized in various autotuning schemes, e.g. with algebraic control design. A program system for automatic estimation, design and simulation was developed.

Keywords—Autotuning, Relay experiment, Limit cycle oscillations, Biased and unbiased relay, Hysteresis, Describing function.

I. INTRODUCTION

The Åström and Hägglund relay feedback test [1] started in 1984 an important tool for automatic controller tuning because it identifies two main parameters for the Ziegler-Nichols method [3]. Previously, relay was mainly used as an amplifier or as a relay back control. The Åström-Hägglund test is based on the observation, when the output lags behind the input by $-\pi$ radians, the closed loop oscillates with a constant period. Then, the ultimate gain and frequency are identified by a simple symmetrical relay feedback experiment proposed in [1]. From the critical values the controller setting was applied by the Ziegler-Nichols rule which is simple but it suffers from several drawbacks.

From that time, many studies have been reported to extend and improve both, the relay feedback experiment as well as tuning and control design principles; see e.g. [2] - [4], [9], [14]-[17]. Many of them need an estimation of transfer function parameters and the original approach provides no explicit parameters of the identified transfer function. During the period of almost three decades, the direct estimation of transfer function parameters instead of critical values began to appear. The extension in relay utilization was performed in e.g. [8] - [11], [21] by an asymmetry and hysteresis of a relay. Nowadays, almost all commercial industrial PID controllers provide the feature of autotuning.

This paper brings a study how the asymmetry and hysteresis influence of the quality and accuracy of identification process. Also the length of the experiment and the relay amplitude can influence the quality of the estimation.

Probably Luyben in [5] was the first who used the approximate describing function (DF) method to estimate the process transfer function from limit cycle measurements.

The main scheme for the relay estimation and/or identification is depicted in Fig.1.

Fig. 1 relay based identification

The goal of the original test was to indicate the critical point in the Nyquist curve of the open loop. However, there are other relays used in identification experiments, e.g. the biased (asymmetrical) relay, main two position symmetrical and asymmetrical (biased) relay without and with hysteresis characteristic are depicted in Fig. 2. A biased (asymmetrical) one characteristic is obtained by a simple vertical moving by an asymmetry shift. Also, the relay without hysteresis is obtained by putting $\epsilon = 0$.

Fig. 2 types of relay
Many research works have been done to improve and refine the effect of fundamental harmonic by using different shapes and structures of the relay element, see [6], [7], [18] - [20]. A limit cycle oscillation for a stable system with positive steady state gain with a biased relay is shown in Fig. 3.

![Fig. 3 biased relay oscillation of stable processes](image)

The critical gain is then given by the relation (see e.g. [11])

\[ r_u = \frac{4}{\pi} \cdot \frac{h_0}{\sqrt{a_r^2 - \varepsilon^2}} \quad \varepsilon = 0 = \frac{4}{\pi} \cdot \frac{h_0}{a_r} \]  

(3)

and the ultimate period \( T_u \) can be read according to Fig. 4.

**B. Unbiased relay experiment**

The relay feedback experiment according to Fig. 1 yields stable harmonic oscillations, i.e. it causes rise of the stable limit cycles (Fig. 3). The describing function method ([15], [11], [13], [24]) is a tool for verification the limit cycle rise. The describing function of the relay \( N(a) \) is considered as a complex gain which depends on the harmonic oscillation amplitude \( a \) and angular frequency \( \omega \) in the relay input \( e(t) \)

\[ e(t) = a \sin \omega t \]  

(4)

The frequency transfer function \( G(j \omega) = A_P(\omega) e^{j \phi_P(\omega)} \) for the first order system (1) gives

\[ A_P(\omega) e^{j \phi_P(\omega)} = -\frac{1}{N(a)} \quad \text{for} \quad 0 \leq \varepsilon \leq a \]  

(6)

otherwise \( N(a) = \infty \). Values \( A(a) \) and \( \phi_N(a) \) represent the critical magnitude and critical characteristic phase, respectively

\[ A_N(a) = \frac{\pi a}{4h_r} \]  

(7)

\[ \phi_N(a) = \arctg \frac{\varepsilon}{\sqrt{a^2 - \varepsilon^2}} - \pi \]

The frequency transfer function \( G(j \omega) = A_P(\omega) e^{j \phi_P(\omega)} \) for the first order system (1) gives

\[ A_P(\omega) = \frac{K}{\sqrt{T^2 \omega^2 + 1}} \]  

(8)

\[ \phi_P(\omega) = -\arctg \omega T - \omega \Theta \]

Comparing \( A_N(a) = A_P(\omega) \) and \( \phi_N(a) = \phi_P(\omega) \) in (7) and (8) gives two equations for the calculation of \( T \) and \( \Theta \). The final relations for the time constant and time delay terms for
FOPDT (1) are given by:

\[ T = \frac{T_v}{2\pi} \sqrt{\frac{16 - K^2 \cdot u_i^2}{\pi \cdot a^2 - 1}} \]

\[ \Theta = \frac{T_v}{2\pi} \left[ \pi - \arctg \left( \frac{2\pi T}{2\pi T} - \arctg \frac{\varepsilon}{\sqrt{a^2 - e^2}} \right) \right] \]

where \( a \) and \( T_v \) are depicted in Fig.1 and \( e \) is hysteresis.

The second order system SOPDT (2) are estimated by relations

\[ T = \frac{T_v}{2\pi} \sqrt{\frac{4 - K^2 \cdot u_i^2}{\pi \cdot a^2 - 1}} \]

\[ \Theta = \frac{T_v}{2\pi} \left[ \pi - 2\arctg \left( \frac{2\pi T}{2\pi T} - \arctg \frac{\varepsilon}{\sqrt{a^2 - e^2}} \right) \right] \]

Relations (9), (10) represent a suitable identification tool for computing time and time delay terms but a relay unbiased experiment is not able to estimate the gain of the controlled system.

\[ T_{a1} = T \cdot \ln \left( \frac{2 \cdot \mu \cdot K \cdot e^\frac{\alpha}{T_v} + \mu_0 \cdot K - \mu \cdot K + \varepsilon}{\mu \cdot K + \mu_0 \cdot K - \varepsilon} \right) \]

\[ T_{a2} = T \cdot \ln \left( \frac{2 \cdot \mu \cdot K \cdot e^\frac{\alpha}{T_v} - \mu_0 \cdot K - \mu \cdot K + \varepsilon}{\mu \cdot K - \mu_0 \cdot K - \varepsilon} \right) \]

The normalized dead time of the process \((L=O/T)\) is obtained from (14) or (15) in the form (see e.g. [8]):

\[ L = \ln \left( \frac{\mu_0 + \mu \cdot K - \varepsilon}{\mu_0 + \mu \cdot K - A_u} \right) \]

or

\[ L = \ln \left( \frac{\mu_0 - \mu \cdot K - \varepsilon}{\mu_0 - \mu \cdot K + A_u} \right) \]

Next, the time constant can be computed from (4) or (5) by solving these formulas:

\[ T = T_{a1} \left( \ln \left( \frac{2 \cdot \mu \cdot K \cdot e^\frac{\alpha}{T_v} + \mu_0 \cdot K - \mu \cdot K + \varepsilon}{\mu \cdot K + \mu_0 \cdot K - \varepsilon} \right) \right)^{-1} \]

or

\[ T = T_{a2} \left( \ln \left( \frac{2 \cdot \mu \cdot K \cdot e^\frac{\alpha}{T_v} - \mu_0 \cdot K - \mu \cdot K + \varepsilon}{\mu \cdot K - \mu_0 \cdot K - \varepsilon} \right) \right)^{-1} \]

and a time delay term is \( \Theta = T \cdot L \).

C. Biased relay experiment

Asymmetrical relays with or without hysteresis bring further progress, see e.g. [2], [6], [7], [13], [21], [22]. After the relay feedback test, the estimation of process parameters can be performed. A typical data response of such relay experiment is depicted in Fig.5. The relay asymmetry is required for the process gain estimation (11) while a symmetrical relay would cause the zero division in the appropriate formula. In this paper, an asymmetrical relay with hysteresis was used. This relay enables to estimate transfer function parameters as well as a time delay term. The proportional gain can be computed by the relation [11]:

\[ K = \frac{\int y(t)dt}{\int u(t)dt}, \quad i = 1, 2, 3, \ldots \]

when the asymmetric relay is used for the relay feedback test, it is shown in Fig.5, the output \( y \) converges to the stationary oscillation in one period. These oscillations are characterized by equations (see [8]):

\[ A_u = \left( \mu_0 + \mu \cdot K \right) \left( 1 - e^{-\frac{\alpha}{T_v}} \right) + \varepsilon \cdot e^{-\frac{\alpha}{T_v}} \]

\[ A_d = \left( \mu_0 - \mu \cdot K \right) \left( 1 - e^{-\frac{\alpha}{T_v}} \right) - \varepsilon \cdot e^{-\frac{\alpha}{T_v}} \]

III. EXAMPLES AND DISCUSSION

The relay tests mentioned in the previous section were applied for explicit estimation transfer functions (1), (2). All simulations were performed in the Matlab, Simulink environment by a program, which the main window shown in Fig. 6. This program was described e.g. in [15], [23] and it is aimed for relay experiments as well for control design and control simulations. At the beginning of the simulation, the
controlled transfer function is defined and parameters for the relay experiment must be entered. Then, a relay experiment is performed and it can be repeated with modified parameters if necessary. After the experiment, parameters of the estimated transfer function are calculated automatically and controller parameters are generated after pushing of the appropriate button, details can be found in [15], [17], [23].

In this contribution, the main emphasis was laid on the accuracy of estimated parameters. The aim is to conclude how to set relay parameters and to give some recommendations. Since the identification relations have to estimate both, time constant as well as a system gain. The time parameters are estimated by a symmetrical relay, while the gain is estimated by a biased relay experiment. Then a contradictory question is concluded: How to utilize a biased relay experiment for estimation of all identified parameters in (1) and (2). The main aim of the research work was to investigate how a biased relay can be used with satisfactory accuracy and how to set up the relay experiment.

The first test transfer function for the first order system is given

\[ G(s) = \frac{3}{4s + 1} e^{-6s} \tag{20} \]

Many relay feedback experiments were performed by the simulation program and the following sensitivity was investigated. The accuracy of estimated parameters depends on main parameters of the relay, namely:

- asymmetry
- hysteresis
- relay amplitude

Table I shows the influence of the asymmetry of the relay on the accuracy of estimation. All entries of Table 1 are in differences between the true and estimated values in %. The upper value of the relay output was 0.30.

<table>
<thead>
<tr>
<th>Asymmetry [%]</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ K [%]</td>
<td>1.7</td>
<td>1.3</td>
<td>1.3</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Δ T [%]</td>
<td>3.5</td>
<td>2.8</td>
<td>2.1</td>
<td>1.2</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>Δ Θ [%]</td>
<td>1.2</td>
<td>1.0</td>
<td>0.8</td>
<td>0.5</td>
<td>0.7</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table I estimation accuracy based on relay asymmetry.

Table II summarizes the sensitivity of the relay hysteresis for transfer function (20). All entries are for comparison in numerical values. The upper relay output was 1.2 lower value - 1.08.

<table>
<thead>
<tr>
<th>Hysteresis ε</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain K</td>
<td>2.88</td>
<td>2.95</td>
<td>2.95</td>
<td>2.92</td>
</tr>
<tr>
<td>Time constant T</td>
<td>3.70</td>
<td>3.88</td>
<td>3.78</td>
<td>3.78</td>
</tr>
<tr>
<td>Time delay Θ</td>
<td>6.12</td>
<td>6.05</td>
<td>6.07</td>
<td>6.12</td>
</tr>
</tbody>
</table>

Table II estimation accuracy based on relay hysteresis

In a similar way, according to Table 2 also a set of experiments were for various values of the lower relay output - 0.96, 0.84, 0.72, 0.60, respectively. The following observations and recommendations can be drawn from the obtained analysis:

- bigger values of asymmetry up to 40% caused better accuracy of all parameters
- better accuracy was achieved for smaller values of hysteresis ε = 0.1; 0.2
- values of relay outputs have no relevant influence on the estimation accuracy

The recommended values for a relay experiment were used for the estimation of the higher order system:

\[ G(s) = \frac{5}{(s + 1)^3} e^{-3s} \tag{21} \]

The relay parameters with ε = 0.1; asymmetry 40% with upper and lower relay outputs 0.30 and -0.18 were used. The resulting first order estimation takes the form:

\[ G(s) = \frac{4.97}{3.58s + 1} e^{-0.19s} \tag{22} \]

Comparison of both step responses of systems (21) and (22) is depicted in Fig. 7. Other results of estimation and autotuning control can be found in [14] - [17], [23].
Fig. 7 step responses of systems (21) and (22)

IV. CONCLUSION

The paper describes main methods for parameter estimation by a feedback relay experiment. The proper and accurate parameter identification plays a key role for a control design, especially in autotuning utilization. Various relay improvements and utilization for control design can be found in [12], [13], [17], [25], [23]. The goal of the paper is to investigate how the estimation is sensitive on the relay settings. Main relay characteristics as asymmetry, hysteresis and amplitude can be recommended for the correct adjustment for the relay experiment.

REFERENCES


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