A comparison of PSO and BFO applications for the PID controller synthesis in time-delay systems

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Abstract. Time delay systems are very common in industry. Their study has been in the focus of worldwide research, taking into consideration the problems associated with their difficult control. PID controllers have found wide application in the control of time delay processes. The classical approaches for obtaining the $K_p$, $K_i$ and $K_d$ parameters of these controllers usually result in overshoot, and significant rising and settling times. In this paper we have proposed the application of PSO and BFO intelligent algorithms for obtaining the optimal parameters of a PID controller, applied in the control of a high order process with time delay. The performance of the proposed control system with PSO and BFO algorithms is analyzed through time response characteristics. A comparison of the proposed approaches, with the cases where integral performance indexes are used to determine the PID parameters, is also introduced. From the obtained results, we conclude that, when applied to time delay processes, the intelligent algorithms achieve better control performance than classical methods.

Keywords: Integral performance index (IAE, ISE, ITAE, ITSE), PID controller, PSO algorithm, BFO algorithm.

1 Introduction

Delays are usually present in control systems as computing or processing delays or as delays in information acquiring. Time delays are common in industrial processes which are characterized by energy and materials’ transport, such as chemical, biological, information, and also measuring, and computing processes. Time delays introduce problems in process control due to decrease of robustness and performance deterioration, which bring the systems close to instability. To achieve the control of such processes, PID controllers have found wide application. Given that approximately 95% of the control schemes in practice are built on PID controllers, finding the right
parameters that improve at maximum the control performance, poses a challenge in itself. Their popularity is related to the fact that they are simple to understand and to operate by operators, and are effective and robust in control.

There exist many methods for the calculation of the PID optimal parameters, in order to obtain a specific characteristic of process time response. To check the effectiveness of various methods for PID controllers, the comparison is usually made by analyzing the transient characteristics of the system.

The characteristics obtained by tuning the PID parameters, often do not meet the control performance criteria defined by the designer. For this reason, latest research is focused on optimization methods based on intelligent algorithms, which result very efficient in solving difficult optimization problems.

Algorithms, inspired by characteristics and organized behaviors of organisms and microorganisms in nature, have recently achieved an increasing interest. Among the algorithms that have been inspired by nature, the most common are particle swarm optimization (PSO) and bacterial foraging optimization (BFO) algorithms. These two optimization methods are the main focus of this work and our proposal is to apply them in finding the optimal PID parameters, in a high order control system with time delay.

Integral of absolute error (IAE), integral of squared error (ISE), integral of time multiplied absolute error (ITAE) and integral of time multiplied squared error (ITSE) integral performance criterions are proposed as optimization functions in our case. These performance indexes will be used to obtain the coefficients of PID controller, and the process transient responses will be analyzed and compared with the methods of obtaining PID coefficients from PSO and BFO algorithms.

The structure of the article is as follows. Section 2 presents the proposed control schemes for the high-order process with time delay, transient response measures and also the integral performance indexes used as optimization (cost) functions to find the coefficients of PID controllers. In section 3, PSO and BFO algorithms are treated, and their application in finding the coefficients of PID controllers. The process that will be considered for various simulations with classical methods (IAE, ISE, ITAE, ITSE) and intelligent methods (PSO and BFO) is presented in section 4. Conclusions obtained from simulations are presented in section 5. Algorithms and computational simulations are performed in MATLAB R2013b environment.

2 Optimization Functions

2.1 Control Scheme

The proposed control scheme for finding the optimal coefficients of PID controller is illustrated in Fig. 1.

Signals presented in the control scheme are:
- $R(s)$-reference signal. In this case the reference signal is a step unit function.
- $Y(s)$-output signal of the system
- $U(s)$-control signal
- $E(s)$-error signal. Derived from $E(s)=R(s)-Y(s)$
Proposed PID controller is in its parallel form, provided by the algorithm:

\[ u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t) \]  

where

- \( u(t) \) - control signal in time domain
- \( K_p \) - proportional coefficient, a tuning parameter
- \( K_i \) - integral coefficient, a tuning parameter
- \( K_d \) - derivative coefficient, a tuning parameter
- \( e(t) \) - error signal in time domain

In Fig. 2 is presented the typical PID controller structure in its parallel form. The process that will be studied is a single input-single output (SISO) third order system with time delay, which depicts many processes in industry.

The mathematical model of delay \( e^{-t\tau} \) according to [1] can be approximated by a rational transfer function of the form:

\[ G_i(s) = \frac{1}{\left(1 + \frac{Ls}{i}\right)^i}, \quad i=1,2,\ldots \]  

which is an \( i \)-th order truncation of expression

\[ e^{-t\tau} = \lim_{i \to \infty} \frac{1}{\left(1 + \frac{Ls}{i}\right)^i} \]
2.2 Transient Response Measures

Analysis for the process transient response in time domain is done through performance quantities [2] like:
- Rising time $t_r$: time required for the output of the system to reach 90% of its final value $h(\infty)$.
- Settling time $t_s$: time after which the output remains within ±2% of the final value $h(\infty)$.
- Peak value $h_{\text{max}}$: peak value of the transient response $h(t)$ of the process.
- Peak time $t_{\text{peak}}$: time required for the transient response $h(t)$ to reach the peak value $h_{\text{max}}$.
- Overshoot $M_r$ (%): output value exceeding final, steady-value of the process, expressed in percentage.

Rising and settling times are measures of response speed of the system. Overshoot, peak value, and peak time are related measures to the quality of response.

2.3 Integral Performance Criteria

In classical control methods, the performance of the entire control system can be estimated quantitatively using a single parameter that is the integral quality criterion $J$. This performance index is useful in treating optimization of parameters and obtaining optimal control designs. According to [3], a system is considered as an optimal control system when the system parameters are tuned to achieve an integral criterion ekstremum, which is usually a minimum value. So, integral criterion should always be a positive number or equal to zero $J \geq 0$. The best achievable control system is the system that minimizes this criterion. The general form of the integral performance criterion is:

$$ J = \int_{0}^{T} f(e(t), r(t), y(t), t) dt $$

where $f$ is a function of error, input, output signals and time. The most common integral criterions are:

- Integral of squared error $ISE = \int_{0}^{T} e^2(t) dt$ (5)

where $T$ is a finite time, chosen arbitrarily, in order for the integral to approach the final stabilized value of the system. Generally $T$ is chosen as equal to $t_s$, settling time.

- Integral of absolute error $IAE = \int_{0}^{T} |e(t)| dt$ (6)

- Integral of time multiplied absolute error $ITAE = \int_{0}^{T} te|e(t)| dt$ (7)

- Integral of time multiplied squared error $ITSE = \int_{0}^{T} te^2(t) dt$ (8)
3 Intelligent Algorithms

3.1 PSO Algorithm

PSO is created by Eberhart and Kennedy in 1995 [4], [5]. The method is suitable for solving nonlinear problems. This algorithm is inspired by natural behavior of animals, such as organized behavior of birds in flock for finding food [6], [7]. PSO algorithm operates using a population (called swarm) of a potential candidate solution (called particles). These particles move around a search space according to a specific routine (law). Movements of particles are guided by their best known position in the search space and also the best known position by the whole flock. When better positions are detected, these positions guide the further movement of the particles. The process is repeated until a satisfactory solution is reached. In this optimization method, a set of particles are placed in a $d$-dimensional space with a random specified speed and position. The initial position of the particle is taken as the best position in the beginning and then particle speed is reassessed based on the experience of other particles of the flock (population).

From [8], PSO algorithm elements are:

- the $i$-th particle in the population represented by:
  \[ x_i = (x_{i1}, x_{i2}, x_{i3}, \ldots, x_{id}) \]  
  \[ (9) \]

- previous best positions of the $i$-th particle are represented by:
  \[ P_{opt} = (P_{opt,1}, P_{opt,2}, P_{opt,3}, \ldots, P_{opt,d}) \]  
  \[ (10) \]

- the index of the best particle in the swarm is $G_{opt}$

- the speed of the $i$-th particle is represented by
  \[ v_i = (v_{i1}, v_{i2}, v_{i3}, \ldots, v_{id}) \]  
  \[ (11) \]

- reassessed speed and distance from $P_{opt,d}$ in $G_{opt,d}$ is given by the law:
  \[ v_{im}^{(t+1)} = W \cdot v_{im}^{(t)} + C_1 \cdot \text{rand()} \left(P_{opt,m}^{(t)} - x_{im}^{(t)}\right) + C_2 \cdot \text{rand()} \left(G_{opt,m}^{(t)} - x_{im}^{(t)}\right) - x_{im}^{(t+1)} \]  
  \[ (12) \]

for $i=1,2,3,\ldots,n$ ; $m=1,2,3,\ldots,d$

where $m$ number of particles in swarm, $d$ dimension index, $t$ iteration index, $v_{im}^{(t)}$ the particle speed in iteration $i$, $W$ weighting factor of inertia, $C_1, C_2$ acceleration constants, $\text{rand()}$ random number between $0$ and $1$, $x_{im}^{(t)}$ actual position of the $i$-th particle in iteration, $P_{opt}$ best previous position of the $i$-th particle, $G_{opt}$ the best particle among all particles of the population.

The flowchart of PSO algorithm is illustrated in Fig. 3.
3.2 BFO Algorithm

BFO is based on research conducted by K.M. Pasino [9], [10], related to development and behavior of E.coli bacteria. In this optimization method there are four typical behaviors that imitate nature [11]:

1) Chemotaxis This process resembles the movement of a bacterium (E.coli) through swimming and displacement via flagella. Biologically an E.Coli can move in two different ways. It can swim for a period of time in the same direction, or it can move alternatively between two modes of movements throughout lifetime. To represent a shift we use a casual direction with a given unit size by $\theta(j)$. This presentation is used to determine the movement direction after a displacement. In particular: $\theta(j+1,k,l) = \theta(j,k,l) + C(i) \cdot \theta(j)$, where $\theta(j,k,l)$ represents the $i$-th bacterium in the $j$-th chemotaxis, the $k$-th
reproduction, the $l$-th elimination and dispersal step $C(i)$ is the size of the step taken in a random direction specified by the unit size $\theta(j)$.

2) **Swarming** E.coli bacteria organize themselves into well-structured colonies with high environmental adaptability using a complex communication mechanism. To create the colonies, bacteria produce signals which are attractive to each other. Analytical presentation of this process is:

$$ J_{ce} = (\theta, P(j, k, l)) = J_{ce}^i(\theta, \theta^i(j, k, l)) = \sum [D_{attractant} \cdot e^{(-W_{attractant} \sum (\theta_m - \theta_m')^2)}] + $$

$$ + \sum [H_{repellant} \cdot e^{(-W_{repellant} \sum (\theta_m - \theta_m')^2)}] $$

where $J_{ce}(\theta, P(j, k, l))$ is the value of the optimization function (to be minimized) to represent a cost function that depends on time. $S$ is the total number of bacteria; $P$ is the number of parameters to be optimized, which are present in each bacteria and $D_{attractant}, W_{attractant}, H_{repellant}, W_{repellant}$ are different coefficients that should be chosen carefully.

3) **Reproduction** Less healthy bacteria die and each healthier bacteria split into two daughter bacteria, each located in the same position.

4) **Elimination and Dispersal** In the local environment, it is possible that the bacteria life of a population can change gradually (e.g. through the consumption of nutrients) or abruptly from other influences. It may happen that in a zone all the bacteria die, or a group is dispersed in a new environment. They can destroy the progress of chemotactic effect, but they can also help the effect, if dispersal occurs in areas with good food sources. From a broader perspective, elimination and dispersal are part of the motion behavior of the population for long distances.

Flowchart of BFO algorithm [12] is illustrated in Fig. 4.

### 4 Simulation Results

In this study we have taken a third order process with time delay, which has a characteristic with many oscillations.

$$ G(s) = \frac{l}{0.31s^4 + 1.75s^2 + 3s} e^{-3s} $$

(14)

As shown in section 2.1, the delay in time will appear in a rational function form:

$$ G_{delay,1}(s) = e^{-3s} = \frac{l}{3s + 1} $$

(15)

The transfer function to be considered during the simulations is:

$$ G(s) = \frac{l}{0.93s^4 + 5.56s^2 + 10.75s + 3s} $$

(16)

In PSO and BFO algorithms that realize the minimization of the integral performance indexes in cost function form, the PID parameters are used as input values and as output is used the optimization value of the PID controller model (17).
Initialize all variables. Set all loop counters and bacterium index $i$ equal to 0.

Increase elimination-dispersal loop counter $l = l + 1$.

Perform elimination-dispersal (for $i = 1, 2, ..., S$) with probability $P_{ed}$, eliminate and disperse to a random location.

Perform reproduction (by eliminating the worse half of population with higher cumulative health and splitting the better half in two).

Increase reproduction loop counter $k = k + 1$.

Increase Chemotactic loop counter $j = j + 1$.

Declare final values $f_{opt}$, $K_p$, $K_i$, $K_d$.

**Fig. 4.** Flowchart of BFO algorithm.

$$\text{Function } [J] = \text{integral criteria } (K_p, K_i, K_d) \quad (17)$$

In the proposed control scheme (Fig. 1), it is intended to tune the three coefficients of PID controller, in order to obtain the best output results, or otherwise said, is intended to optimize the PID coefficients to achieve optimal results. Integral criterions used as cost functions, evaluate the performance of various combinations of PID coefficients in a 3-dimensional search domain. Each point in this 3-dimensional search domain for the proposed algorithm, represents a certain combination of $[K_p, K_i, K_d]$ coefficients, for which a certain transient response of the system is achieved.

4.1 Classical Approach

Using IAE, ISE, ITAE, ITSE integral criterions, treated in Section 2.3, in Fig. 5 are obtained the transient responses of our process. Executing the algorithms [13], we
obtain the corresponding coefficients of PID controller for the four performance
criterions. PID controller coefficients, obtained by the classical algorithms are shown
in Table 1.

Table 1. PID controllers obtained by classical algorithms.

<table>
<thead>
<tr>
<th>Performance Index</th>
<th>Coefficients of PID controllers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( K_p )</td>
</tr>
<tr>
<td>ISE</td>
<td>3.042</td>
</tr>
<tr>
<td>IAE</td>
<td>2.328</td>
</tr>
<tr>
<td>ITSE</td>
<td>3.505</td>
</tr>
<tr>
<td>ITAE</td>
<td>8.596</td>
</tr>
</tbody>
</table>

Fig. 5. Transient responses for classical algorithms.

4.2 Intelligent PSO and BFO algorithms approach

In order to find the PID controllers coefficients by the intelligent algorithms PSO
and BFO, in this study we have used the 3-dimensional search domain where the \( K_p \),
\( K_i \), \( K_d \) values are the three dimensions of domain. In both algorithms, the four integral
criterions of time domain, treated in Section 2.3, were chosen as optimization
functions. The algorithms were executed in Matlab R2013b environment where as
cost functions were used:
- ISE: \( J=e^*e^*dt \)
- IAE: \( J=sum(abs(e)^*dt) \)
- ITSE: \( J=(t.*e^*dt)^*e; \)
- ITAE: \( J=sum(t.*abs(e)^*dt) \)
The number of computing iterations that the algorithms will perform, is based on the
calculations duration and complexity of the optimizing problem. The initial constants
for the computing PSO algorithm were taken \( W=0.3, C_1 = C_2=1.5 \). At the end of PSO
algorithm execution, at Matlab prompt are displayed the \([K_p, K_i, K_d]\) items of search
domain, which is the final point of global optimum noted as $g_{opt}$, that corresponds to the minimum value of cost function, noted as $f_{opt}$.

Coefficients of PID controllers, obtained by intelligent PSO algorithm are shown in Table 2. In Table 2 are also shown the best (minimum) values of cost functions (integral criterions).

Table 2. PID controllers obtained by PSO algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Coefficients of PID controllers</th>
<th>$f_{opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO-ISE</td>
<td>$K_p=0.8587$, $K_i=0.3290$, $K_d=18.7447$</td>
<td>0.7002</td>
</tr>
<tr>
<td>PSO-IAE</td>
<td>$K_p=3.4373$, $K_i=0.3114$, $K_d=14.1379$</td>
<td>1.6575</td>
</tr>
<tr>
<td>PSO-ITSE</td>
<td>$K_p=3.6658$, $K_i=0.1784$, $K_d=9.2223$</td>
<td>0.6179</td>
</tr>
<tr>
<td>PSO-ITAE</td>
<td>$K_p=5.2616$, $K_i=0.3611$, $K_d=11.2419$</td>
<td>5.1510</td>
</tr>
</tbody>
</table>

The constants used for the initialization of the computing program in BFO algorithm are taken $D_{attractant}=0.01$, $W_{attractant}=0.01$, $H_{repellant}=0.01$, $W_{repellant}=0.01$. Table 3 presents the values of PID controller coefficients obtained by BFO algorithm, and the corresponding minimum values of cost functions.

Table 3. PID controllers obtained by BFO algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Coefficients of PID controllers</th>
<th>$f_{opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BFO-ISE</td>
<td>$K_p=1.5823$, $K_i=1.5187$, $K_d=17.8117$</td>
<td>0.7865</td>
</tr>
<tr>
<td>BFO-IAE</td>
<td>$K_p=8.1119$, $K_i=0.4878$, $K_d=8.7152$</td>
<td>1.9695</td>
</tr>
<tr>
<td>BFO-ITSE</td>
<td>$K_p=4.3073$, $K_i=0.8238$, $K_d=6.7563$</td>
<td>1.7230</td>
</tr>
<tr>
<td>BFO-ITAE</td>
<td>$K_p=5.7679$, $K_i=0.7655$, $K_d=7.2006$</td>
<td>4.2287</td>
</tr>
</tbody>
</table>

Tables 4,5,6 present the transient response measures for the three cases.

Table 4. Characteristics of transient responses for classical approach

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Classical approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IAE</td>
</tr>
<tr>
<td>Rising time $t_r$</td>
<td>25.29</td>
</tr>
<tr>
<td>Settling time $t_s$</td>
<td>210.13</td>
</tr>
<tr>
<td>Overshoot $M_r$ (%)</td>
<td>31.82</td>
</tr>
<tr>
<td>Peak value $h_{peak}$</td>
<td>1.32</td>
</tr>
<tr>
<td>Peak time $t_{peak}$</td>
<td>64</td>
</tr>
</tbody>
</table>
Table 5. Characteristics for transient responses for PSO approach

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>IAE</th>
<th>ISE</th>
<th>ITSE</th>
<th>ITAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rising time ( t_r )</td>
<td>0.83</td>
<td>0.72</td>
<td>1.09</td>
<td>0.89</td>
</tr>
<tr>
<td>Settling time ( t_s )</td>
<td>21.39</td>
<td>102.42</td>
<td>17.45</td>
<td>12.17</td>
</tr>
<tr>
<td>Overshoot ( M_r(%) )</td>
<td>22.2</td>
<td>20.52</td>
<td>18.22</td>
<td>30.27</td>
</tr>
<tr>
<td>Peak value ( h_{\text{peak}} )</td>
<td>1.22</td>
<td>1.21</td>
<td>1.18</td>
<td>1.30</td>
</tr>
<tr>
<td>Peak time ( t_{\text{peak}} )</td>
<td>1.97</td>
<td>1.67</td>
<td>2.59</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Table 6. Characteristics for transient responses for BFO approach

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>IAE</th>
<th>ISE</th>
<th>ITSE</th>
<th>ITAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rising time ( t_r )</td>
<td>0.73</td>
<td>0.91</td>
<td>1.19</td>
<td>1.07</td>
</tr>
<tr>
<td>Settling time ( t_s )</td>
<td>100.68</td>
<td>9.26</td>
<td>7.62</td>
<td>10.87</td>
</tr>
<tr>
<td>Overshoot ( M_r(%) )</td>
<td>24.04</td>
<td>53.12</td>
<td>38.7</td>
<td>46.18</td>
</tr>
<tr>
<td>Peak value ( h_{\text{peak}} )</td>
<td>1.24</td>
<td>1.53</td>
<td>1.39</td>
<td>1.46</td>
</tr>
<tr>
<td>Peak time ( t_{\text{peak}} )</td>
<td>1.75</td>
<td>2.59</td>
<td>3.44</td>
<td>3.06</td>
</tr>
</tbody>
</table>

Fig. 6 presents the process transient responses obtained by PSO algorithm for various optimization functions.

![Step response of the process](image)

**Fig. 6.** Transient responses for PSO algorithms.

Fig. 7 presents the process transient responses obtained by BFO algorithm, for various optimization functions.
Fig. 7. Transient responses for BFO algorithms.

Fig. 8. presents the best transient responses of process obtained by ITSE, PSO-ITSE, BFO-ITSE algorithms.

5 Conclusions

Based on the performed simulations, we arrive at the conclusion that methods based on PSO and BFO intelligent algorithms are quite efficient in achieving a very good control of processes with time delay. Specifically, the resulting rise time $t_r$ and settling time $t_s$ are reduced further, resulting in control systems that have a faster response to changes at the system’s input. From Tables 4, 5, 6, we conclude that ITSE integral criterion, used as a cost function to find the optimal values of $K_p$, $K_i$, $K_d$ coefficients of PID controller is the best function that can be used for the methods discussed above. Comparing the results from the application of intelligent algorithms, we concluded that the PSO algorithm is more efficient and provides a better tuning of the process. It provides the best value (minimum value) of the cost function $f_{opt}$. PSO
algorithm has simpler computing architecture than BFO algorithm, resulting in a faster algorithm in computing time.

References