A Model of the Universe According to the Virial Theorem

Hasan Arslan

Abstract— The model of the universe is defined according to the virial theorem. The previous works are searched. The Energy of the universe and the distance of it are related to each other by an equation derived from the virial theorem. The universe is considered to be under a periodic motion.

Keywords—Virial theorem, Hubble's constant, Accelerating universe, Model of the Universes.

I. Introduction

The article is designed as follows. The derivation of the virial theorem is given in section II both in classical case and in quantum mechanical case according to the previous works [1-7]. In section III, the model of the universe is expressed. The conclusion is given in section IV.

II. The Virial Theorem

In the references [1-7] the virial theorem is derived as follows:

$$G = \sum_i \vec{p}_i . \vec{r}_i \tag{1}$$

here G is considered to be a quantity, product of the momentum and the position of the particle in a stable system. Taking the derivative of Equation (1), we get:

$$\frac{dG}{dt} = \sum_i \left(\frac{d\vec{p}_i}{dt} . \vec{r}_i + \vec{p}_i . \frac{d\vec{r}_i}{dt} \right) \tag{2}$$

The second term on the right hand of the equation (2) can be written as [4, 5]

$$\sum_i \vec{p}_i \frac{d\vec{r}_i}{dt} = \sum_i \left(m\dot{\vec{r}}_i \right) \dot{\vec{r}}_i = m\dot{r}_i^{\,2} = 2T \quad \text{and} \sum_i \dot{\vec{p}}_i r_i = \sum_i F_i r_i \tag{3}$$

Here T is the kinetic energy. If the quantity G is bounded in a time interval, one can write:

$$\frac{1}{\tau} \int_0^\tau \frac{dG}{dt} dt = \frac{1}{\tau} \left(G(\tau) - G(0) \right) = 0 \tag{4}$$

From Equation (2), it can be written as

$$\frac{1}{\tau} \int_0^\tau \frac{dG}{dt} dt = 2T + \sum \vec{F}_i . \vec{r}_i \tag{5}$$

for the periodical momentum. From Equations (4) and (5) we obtain;

.
$$2T = -\sum_i \vec{F}_i . \vec{r}_i \tag{6}$$

Equation (6) is the virial theorem in the classical case.

Now, we are looking for the quantum mechanical virial theorem. The time-dependent Schrödinger Equation is;

$$i\hbar \frac{d\psi}{dt} = H\psi . \tag{7}$$

The derivative of expectation value of an operator *A* with respect to time is;

$$i\hbar \frac{d}{dt} \langle \psi | A | \psi \rangle = \langle \psi | [H, A] | \psi \rangle . \tag{8}$$

Let us choose A to be $A = \vec{r} . \vec{p}$ [1-7]. Putting this in Equation (7) and taking *A* to be time-independent, we get;

$$\langle \psi | [H, A] | \psi \rangle = 0 \tag{9}$$

Then virial theorem is obtained as

$$\begin{aligned} [H, A] &= \left[\frac{p^2}{2m} + V(r), \vec{r} . \vec{p} \right] \\ &= i\hbar \vec{r} . \nabla V - \frac{i\hbar}{m} \vec{p}^{\,2} \\ &= i\hbar \vec{r} . \nabla V - 2i\hbar T = 0 \end{aligned} \tag{10}$$

where *T* is the kinetic energy and *V* is the potential energy. Then, the virial theorem can be written as

$$2\langle T \rangle = \langle \vec{r} . \vec{\nabla} V \rangle \tag{11}$$

The kinetic energy can be defined by

$$T = \frac{p^2}{2m} \tag{12}$$

where *p* is the momentum operator and *m* is the mass of the particle under consideration. The momentum operator is

$$p = -i\hbar \nabla \tag{13}$$

Using Equation (11) for a potential of the form $V = kr^n$, the kinetic energy is obtained as

$$\langle T \rangle = \frac{n}{2} \langle V(r) \rangle \tag{14}$$

Department of Physics, Bingöl University, Bingöl, Turkey

e-mail: hasanarslan46@yahoo.com

We note here that if $n = 0$, we do not have to write Equation (14) because of the singularity, for $n \neq 0$ the equation is valid.
Due to the Equation (12) and the Equation (14), the following relation can be written:

$$V(r) = \frac{p^2}{nm}. \quad (15)$$

The mechanical energy of a closed system is conserved and is given by;

$$E = T + V \quad (16)$$

From Equation (11), we can write

$$T = \frac{1}{2} r \nabla V \quad (17)$$

Then, we can rearrange this last equation by taking

$$\nabla V = \frac{\partial V}{\partial r} \quad (18)$$

as

$$V = 2T \ln r / r_0. \quad (19)$$

Then the total mechanical energy of a closed system becomes

$$E = (1 + 2\ln r / r_0)T \quad (20)$$

or it can be written for the distance travelled as

$$r = r_0 e^{\frac{(E-T)}{2T}}. \quad (21)$$

Here r_0 is the distance between the two spherical shell orbit of the universe, and r is that distance of it at a later time. For whom like to make a search and to learn more about on the virial theorem, I would like to recommend his/her to look the references [8- 26] and the references given therein.

III. The Construction of the Model of the Universe

The Bing Bang Theory suggest that the universe began to be formed by explotion of a very tiny, high densed, very hot point and then began to be cooled quickly and expanded too fast to the outer dimensions and formed the space of the universe where everything,like galaxies-stars- clusters- planets etc., take place in.

One of the Hubble's major discovery was based on comparing his measurements of the Cepheid-based galaxy distance determinations with measurements of the relative velocities of these galaxies. He showed that more distant galaxies were moving away from us more rapidly with speed v moving away from us as:

$$v = H_o d \quad (21)$$

where d is its distance. The constant of proportionality H_o is now named as the Hubble constant. The common unit of measuring velocity is km/sec, while the most common unit of measuring the distance to the nearby galaxies is Megaparsec (Mpc) which is equal to 3.26 million light years . Thus the units of the Hubble constant is (km/sec)/Mpc[27, 28].

This discovery is the beginning of the modern age of cosmology. Cepheid variables remain one of the best methods of measuring distances to galaxies and these variables are very important to determine the expansion rate and the age of the universe [28]. Also, one who would like to learn more about the Hubble's constant can see the studies[27- 36].

Now, the kinetic energy can be written in terms of the Hubble's constant as

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m H_0^2 r^2 \quad (22)$$

where r is the distance from the inner spherical orbit of the universe to the outer spherical orbit of the universe, m is the mass included in this universe. Then, the total energy of the universe as a closed system is written as

$$E = \frac{1}{2} m H_0^2 r^2 (1 + 2\ln r / r_0) \quad (23)$$

Here, it is considered the universe as an closed system, and the energy of it is given by the Equation (23). In the model, the universe is expanding out through an undestroyable wall, and there are other universes between the undestroyable wall and the universe under consideration because our universe is expanding. A model of the universe is defined by Hawking as "the universe in a nutshell" [37]. Undestroyable wall is assumed to be a region with a too high energy which allow nothing to pass through or effect it in any situation. I would like to describe the motion of the universe we inside in as from the beginning of the Big Bang to outward direction till this undestroyable wall. Then it has to have an inward motion through to make ready the conditions of a new Big Bang explosion in outward direction again. Here the Big Bang can be considered in two stages. First, the Big Bang of the innermost Universe in the outward direction. Second, the Bing Bang of the outermost Universe in inward direction. And also, if our universe is expanding, then there are some of the universes outside our universe that are shrinking [37, 38], and since our universe is accelerating then there are other expanding universes inside it. The undestroyable wall covering these universes is the outer most region of them.

To simplify my model, I would like to say that the motions of the universes are like the motion of valve plungers of a vehicle, when some of them are in upward motion, the others are in downward motion. Upward motion refers to the expansions of the universes, downward motion to the shrinkage of the others. Or, it can be described as a motion of a spring moving forward and backward about its equilibrium point on a frictionless surface with a mass attached to its end. And, the universes are like the spherical shells inside each other with the regions that themselves are in motion except the outermost region of the outermost universe. The regions of each universe is covered by the two spherical orbits like that of each orbit of the electron move in. These universes are thought to have parallel spheres of shells. And, these parallel spherical shells are inside each other, one covered by the other. The parallel universes are also studied in [39-41].

IV. CONCLUSION

If the universes are closed or stable systems, the energy of the universes and the distances of them are related to each other for each universe by the Equation (23) derived from the virial theorem. Therefore, if they are the closed or stable systems, this equation should describe the motions of the universes. As a result, since the virial theorem describes the periodic motion of the closed systems, we can take the beginning of the Big Bang as the initial time of this periodic motion.

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Hasan Arslan was born in Elbistan, in 1966. He graduated from METU Physics Department with BS degree in 1993, in Ankara, Turkey. He got Msc degree from Çukurova University Physics Department in 1998, PhD degree in the same Institution in 2008 with the completed related studies in Theoretical High Energy Physics in Adana, Turkey.

Hasan worked as an English, Maths, Science teacher in private schools in Adana-Turkey in 1994. He worked as an official teacher of English in National Education Ministry of Turkey from 1998 to 2009 in Adana-Turkey. He has been working in Bingöl University Physics Department from 2009 till now. Some of his works can be listed as:

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