Computing Non-Hydrostatic Pressure on Flip Buckets by Processing NASIR Finite Volume Solver Results

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Abstract

Due to the centrifugal force, pressure distribution is non-hydrostatic in flip bucket spillways. In this work, two dimensional modeling of supercritical flow parameters which is computed by NASIR software and local curvature are used for post-processing and calculation of dynamic pressure on every grid points of the bucket bed. The module of the utilized software solves Shallow Water Flow-solver adopted for steep slopes on the unstructured triangular meshes using finite volume method and computes flow depth and depth average velocities. The curvature is modeled using geometrical features of the unstructured triangular mesh which is utilized for modeling the three-dimensional bed surface. The geometrical modeling of the bed surface curvatures is utilized for estimating the vertical curvatures on the nodal points of the computational mesh. An analytical relation which utilizes bed curvature value is applied for calculating the vertical distribution of local dynamic pressure at the mesh nodes. This simulation strategy is verified by evaluating non-hydrostatic pressure for ending curves of the spillway buckets at the end of supercritical flow water were modeled. The non-hydrostatic pressure results of present modeling strategy are compared with the reported experimental measurements.

Keywords: Flip Bucket, Non-Hydrostatic Pressure, NASIR Solver, Shallow Water Equations
1. Introduction
For design of the flip bucket spillways, pressure distributions at bucket bed as one of the vulnerable parts of a spillway have to be evaluated. The bucket of the spillways is one of the points that sever pressure changes may accrue due to the vertical curvature of the supercritical flow bed and the presence of high velocity flow in the locations with a vertical curve. This condition cause significant centrifugal forces, and consequently, excess positive or negative non-hydrostatic pressure would be developed in these places.

Developments in powerful computer hard-wares and capable software’s have made the numerical simulation as a suitable means for modeling the real world engineering cases. For the flow problems in which variation parameters in current depth is negligible, the horizontal two-dimensional (depth averaged) numerical flow solvers are attractive alternatives [1]. Such a model is practically applicable for supercritical flow in which the depth average value covers most of the profile normal to the bed surface [2].

However, the shallow water equations which are commonly used as the most appropriate mathematical model for horizontal two-dimensional simulation, suffers from two major restriction due to considering mild slope and hydrostatic assumptions.

The mild slope application restriction of horizontal two-dimensional flow solvers is recently relaxed by modification of the shallow water equations for steep slope (in the main flow direction) [3]. Such a model is successfully applied for modeling supercritical flow on some types of chute spillways with variable slope [4]. This modeling strategy, paved the way for post-processing of the air concentrations (entered from the free surface and bottom aerators) [5].

In present work, a post-processing on computed depth averaged flow parameter are proposed for treatment of the non-hydrostatic values of the pressure field in the vicinity of the vertically curved supercritical flow bed. This treatment is performed by utilizing the analytical relations [6] which calculates local centrifugal pressure using the computed depth averaged flow parameters (by a version of NASIR\(^{1}\) software which solves modified Shallow Water Equations for variable steep slope surface [4]) and nodal values of the curvature (on the mesh of the three-dimensional bed surface).

2. Depth Average Mathematical Model for Varying Steep Slopes
Mathematical model used in the depth averaged flow solver module of NASIR software includes shallow water equations corrected for varying steep slope as [4]:

\[
\begin{align*}
\frac{\partial h'}{\partial t} + \frac{\partial (h'u')}{\partial x'} + \frac{\partial (h'v')}{\partial y'} & = 0 \\
\frac{\partial (h'u')}{\partial t} + \frac{\partial (u'h' + v'h' + u'h'' + v'h'')}{\partial x'} + \frac{\partial (v'h')}{\partial y'} & = \frac{\partial}{\partial x'} \left[ h' \frac{gh'}{\cos \alpha} \right] \\
\frac{\partial (h'v')}{\partial t} + \frac{\partial (u'h' + v'h' + u'h'' + v'h'')}{\partial x'} + \frac{\partial (v'h')}{\partial y'} & = \frac{\partial}{\partial y'} \left[ h' \frac{gh'}{\cos \alpha} \right] \\
+ \frac{gh'}{2 \rho_w \cos \alpha} \frac{\partial \rho_m}{\partial x'} & = gh' \sin \alpha - gh' S_{f_{g_{c}}}
\end{align*}
\]

Here

\[
S_{f_{c}} = \frac{n^2 u' \sqrt{u'^2 + v'^2}}{h'^{4/3}} \quad ; \quad S_{f_{j}} = \frac{n^2 v' \sqrt{u'^2 + v'^2}}{h'^{4/3}}
\]

and \(x'\) tangential axis to the bed in slope direction and horizontal \(y\) axis are general coordinates. \(u'\) and \(v\) are velocity components in

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\(^{1}\)Numerical \textit{A}nalyzer for \textit{S}cientific and \textit{I}ndustrial \textit{R}equirements (NASIR)
direction of $x'$ and $y$, $h'$ is flow depth, and $g$ is acceleration due to gravity. $\alpha$ is bed slope angle following $x'$, and $S_{f_x}$ and $S_{f_y}$ are friction slopes in direction of $x'$ and $y$. $n$ is Manning roughness coefficient. $\rho_m$ is water and air mixed density, $\rho_w$ is the pure water density, and $C_{mean}$ is air averaged density. For the cases that water and air mixed density is not considered, term $\frac{gh'^2}{2\rho_w \cos \alpha} \frac{\partial \rho_m}{\partial x}$ and $\frac{gh'^2}{2\rho_w \cos \alpha} \frac{\partial \rho_m}{\partial y}$ will be omitted from the above equations.

3. Finite Volume Formulation of the Depth Average Flow Equations

Unstructured triangular grid and finite volume numerical solution method have been used in this modeling. In order to do this flow equations given in previous part could be written in following vector form:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x'} + \frac{\partial F}{\partial y'} = S ,$$

Where

$$E = \begin{bmatrix} hu'^2 \\
\frac{gh'^2}{2 \cos \alpha} + \frac{\rho_m gh'^2}{2 \rho_w \cos \alpha}
\end{bmatrix}, \quad Q = \begin{bmatrix} h' \\
h'u' \\
0
\end{bmatrix}, \quad S = \begin{bmatrix} gh'(Sin \alpha - S_{f_x}) \\
- gh'S_{f_y}
\end{bmatrix}, \quad \text{and}

$$F = \begin{bmatrix} h'v \\
h'u'v \\
h'v^2 + \frac{gh'^2}{2 \cos \alpha} + \frac{\rho_m gh'^2}{2 \rho_w \cos \alpha}
\end{bmatrix} .$$

If the above set of the equations be placed in general integrated over the control volume area $\Omega$, it can be written as:

$$\int_{\Omega} \left( \frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x'} + \frac{\partial F}{\partial y'} \right) dx'dy = \int_{\Omega} Sdx'dy .$$

The discrete finite volume formulation form of the above equation is written as:

$$Q^{n+1} = Q^n - \frac{\Delta t}{\Omega} \sum_{k=1}^{N} \left( E \Delta x - F \Delta y \right)_k + S \Delta t .$$

In which, $\bar{E}$ and $\bar{F}$ are average flux components at the boundary edge of $k$ of the control volume with $\Delta x$ and $\Delta y$ Cartesian coordinate components, and $N$ is up to the method of solution (cell vertices, cell center and Galerkin methods). It has to be mentioned that $Q^n$ and $Q^{n+1}$ refered to the vectors of the unknown variables at the centre of the control volume in two sequential iterations or time steps of explicit solution procedure.

The above mentioned finite volume formulation can be used for numerical computation of depth-averaged parameter (ie $u'$, $v$ and $h$) at every nodal point of the utilized mesh. Therefore, the hydrostatic pressure can be calculated at any position of the flow depth of each nodal point.

Considering that the pressure has two components of hydrostatic and non-hydrostatic components, the non-hydrostatic component of the pressure should be calculated using computed local velocity at the mesh nodes with curvature in vertical plane.

4. Dynamic Pressure on Curved Beds

Over the flip bucket spillways, hydrostatic pressure assumption is not valid. In such parts, non-hydrostatic pressure (less or more than hydrostatic pressure) may form due to the curvatures in the flow bed. Hence, considering flow’s curvature affect corrections have to be imposed on the hydrostatic pressure. In order to do that in the present research, analytic and
experimental formulas have been used for calculation of local non-hydrostatic pressure.

An analytical relation which is suitable for evaluating the pressure distribution in flows over beds with vertical curves was derived by Chow, as [6]:

\[ h_t = h_s + \frac{V^2 h_s}{gR} \]

Where, \( h_t \) is total local pressure head, \( h_s \) is local hydrostatic pressure head, \( V \) is local velocity (in main flow direction), \( R \) is bed curvature in vertical plane and \( g \) is the gravity acceleration. The above analytical relation is derived considering basic assumption of uniform distribution of velocity and constant curve radii \( R \) along the segment associated with the location [6].

5. Curvature at nodal points of mesh

The relation reviewed in the former section calculates local non-hydrostatic pressure over the vertically curved beds using flow parameters and the bed curvature value.

Local velocity and hydrostatic pressure head which are required in this analytical relation can be obtained from the numerically computed results of depth-averaged flow equations, While bed curvature as a key parameter must be known by a reliable method.

Therefore, the total pressure including hydrostatic pressure and non-hydrostatic pressure head components, can be calculated by post-processing on the computed flow parameters and the curvature on the nodal points of the 3D surface mesh (representing the channel’s bed geometric characteristics).

Local curvature in vertical plane (\( \kappa = 1/R \)) at every point has to be specified prior to the start of the computations. In order to calculate curvature should be computed at all points of a triangular unstructured mesh (which is converted as a 3D surface by assigning the z coordinate for every nodal point to model flow bed geometry).

Although in spillway design, there are common mathematical functions such as specific polynomials or conic curves which are used to define spillway bed geometry at every part, a general geometry modeling of nodal points with arbitrary generation, will be more comprehensive.

In order to consider the local curvature, the 3D surface mesh which is used to solve the depth-average flow equations can be utilized. The grid points of the mesh provide required geometric data in terms of Cartesian coordinate \( X_k(x_k, y_k, z_k), k = 1, 2, 3, ..., n \). The curvature value at any nodal point of the mesh can be estimated by evaluating curvature of the curve fitted through the points.

Since the acceleration component normal to the direction of curvilinear flow is the major cause of non-hydrostatic pressure [6], that is to be calculated is the curvature values of the fitted curve along the main flow direction.

Therefore, application of an appropriate curve fitting method [7] in geometric modeling, to model the longitudinal profiles of the bed surface would be an admissible solution. The curvature obtained by curve fitting on the boundaries of the spillway parallel to the stream can be used for finding the curvature values at the grid points between two boundary lines by interpolation.

It should be noted that, in order to accurate geometry modeling (curve fitting) of the cases with multiple bed slopes and complex geometric features, there may require considering several independent segments. However, the mesh partition facilities of the flow solver for dividing
the mesh zones according to various flow regimes can help resolving this requirement.

6. Evaluation of Modeling Results

In this study, hydro-static pressure at the bucket are modified by developing a post processing for systematic calculations of non-hydrostatic pressure as and adding to the hydrostatic pressure obtained from the results of depth-average flow solver. This post processing considers the supercritical flow parameters and vertical curvature of bed surface, and then, calculates consequent excessive non-hydrostatic pressure using analytical relations. Finally, the static pressure values resulted by flow solver are sum up algebraically, and the outcome would be total pressure.

6.1 Flip bucket at the end of steep slope chute

In this part, reported measurements of an experimental model which was introduced by AAKhan & PM. Steffler (1996)[8], are used for accuracy evaluation of the dynamic pressure computed by the present numerical simulation and post-processing.

The model boundary conditions for these cases are specified upstream depth (h0) and vanishing derivatives of extra pressure and velocity variables. Flow conditions on inlet boundary have been entered within discharge per unit width. As downstream flow is supercritical, no conditions are applied at downstream end.

1.1 Comparison of Modeling Results with Experimental Results

The geometrical dimensions of the spillway are digitally modeled according to the laboratory model. The bottom surface of the spillway is modeled by an unstructured triangular mesh (Figure 1-a). The solve domain is divided to five sections in this case based on the flow regime type as well as the bed slope and curvature. In table (1) domain partitions for systematic calculations of pressure over flip bucket model are presented.

In supercritical region without curvature the flow solver pressure results are acceptable as well as subcritical section. The section which has been chosen to distinguish the error is supercritical part that has curved bed. In figure 14 water surface profiles obtained from the flow solver has been compared with the experimental results. As it is noticeable, numerical results compared to the experimental data have a negligible error.

It is evident that the effect of the bucket curvature on pressure is increasing as it is a concave curve. As can be seen from Figure (2) and Table (2) results obtained from post-processing using experimental equations given by Heller et.al has much less error than using Chow formula. As it is shown in Figure (3), using these experiential equations leads to achieve a smooth distribution for pressure.

According to Table (2) Averaged relative error of bed pressure values obtained by the present modeling (relations of Heller et.al.) for discharges of $q = 0.0187 m^2/s$ and $q = 0.0292 m^2/s$ respectively have been decreased 50% and 58% comparing with the hydrostatic pressure values.

7. Conclusion

In this paper, the hydrostatic pressure computed from a depth average flow solver (which is adopted for steep slopes) is modified by applying an analytic relation for calculation of dynamic pressure at spillway buckets. The analytical relations for calculation of centrifugal pressure force at bucket bed uses local curvature of the flow bed and flow velocity magnitude.
Here, the required local curvature is calculated from the geometric features of the three-dimensional unstructured triangular surface mesh which is used for numerical computation of depth averaged flow parameters (i.e., flow depth and depth averaged velocity components). The required local velocity magnitudes are calculated from the depth averaged velocity components. Finally, the total pressure at every point is calculated by sum of hydrostatic and non-hydrostatic pressures at the nodal points of the computational mesh. The comparison of the computed results with the reported experimental measurements shows promising agreements.

Figure 1 - Bucket profile at the end of slopping chute a) 3D view of unstructured triangular grid b) 3D view of flow depth map obtained from flow solver c) velocity vectors obtained from flow solver.

Table 1 - Domain partitions for systematic calculations of pressure over flip bucket

<table>
<thead>
<tr>
<th>Section No.</th>
<th>Start Coordinates (m)</th>
<th>End Coordinates (m)</th>
<th>Flow Considerations</th>
<th>curvature</th>
<th>Fitted curve</th>
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</thead>
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<tr>
<td>1</td>
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<td>1.15</td>
<td>Subcritical</td>
<td>No Curvature</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>1.15</td>
<td>1.3</td>
<td>Supercritical</td>
<td>Convex Curvature</td>
<td>Conic</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
<td>3.00</td>
<td>Supercritical</td>
<td>No Curvature</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>3.00</td>
<td>3.25</td>
<td>Supercritical</td>
<td>Concave Curvature</td>
<td>Circle</td>
</tr>
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</table>
Table (2) – Comparison of relative error values between numerical models for the bucket at the end of slopping chute

<table>
<thead>
<tr>
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<th>Averaged Relative Error for Pressure (%)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( q = 0.0187 \text{m}^2/\text{s} )</td>
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<tr>
<td>Depth-averaged numerical model</td>
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<td>Corrected hydrostatic pressure (Heller et al formula)</td>
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8. References


