Bayesian channel estimation in chaos based DS-CDMA system

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***Abstract*—This paper proposes maximum a priori (MAP) channel estimation technique in chaos based code division multiple access (CDMA) system. Two different cases are considered for estimating the fading channel. In the first case, channel coefficients are estimated with the help of chaotic sequences. In the second case estimation is performed without including the chaotic sequence in the estimation algorithm. Simulation results shows that the MAP estimation algorithms performance is better for the first case.**

***Index Terms*—Channel estimation, CDMA, Chaotic sequence, Bayesian estimation, MAP**

I. Introduction

Fading is the phenomena which makes wireless communication more difficult as compare to other communication systems e.g. optical fiber communication and wired communication etc. For many wireless systems, independent of whether time division multiple access (TDMA) or code division multiple access (CDMA) is employed, estimation of channel fading coefficient is necessary for high speed communication. Channel estimates can be updated frame by frame for slower fading rate as compare to frame rate. If channel coefficients changes significantly within the frame then it is necessary to update coefficients iteratively based on symbol by symbol basis [1], [2].

Various estimation methods have been studied in last few decades and each method has its own advantages and disadvantages. Minimum mean square estimators (MMSE) [3], [4], [5] are easy to implement and perform well in flat fading environment. But these estimators require correlation computation and have poor performance for time varying channel estimation. Bayesian estimators [6], [7], [8], [9] used prior knowledge of data to generate posterior analysis. Therefore performance extensively depends on prior informations. On the other hand, neural networks [10], [11], [12] do not require prior knowledge of channel statistic, but there is huge computational burden for training process. Finally, particle filters [13], [14], [15], [16] use the sequential Monte Carlo sampling method to implement recursive Bayesian filter. But these filters have very high computational load for correcting each particle, which results in higher energy consumption. Therefore hardware implementation of these filters are difficult.

The chaotic signals generated from the same chaotic map has high auto correlation and very low cross correlation values. Further, these signals are very sensitive to initial conditions, therefore infinite number of chaotic sequences can be generated from a chaotic map. Hence, chaos based CDMA system gains significant interest among the researchers in last decade [17], [18], [12], [19], [20], [21], [22], [23], [24], [25]. Each user in CDMA system is distinguish by it's spreading code. Bayesian estimators i.e. MAP and maximum likelihood (ML) are extensively studied for CDMA systems with binary spreading codes [26], [27], [28], [29], [30], [31]. However, to our best knowledge, performance of these estimators never studied for chaos based CDMA system.

Objective of this research work is to study the Bayesian channel estimator for chaos based CDMA system for downlink communication. MAP estimator equation is derived for these systems, which needs a prior knowledge of channel statistics. Further, we have also derived the ML estimation equation for considering the case where the mean and variance of the channel is unknown at the receiver. Two algorithms are derived to consider the multiplexed pilot-data case and added pilot-data case. In multiplexed pilot-data case, after demultiplexing, channle estimation can be performed directly on the extracted pilot signal. Whereas for added pilot-data case, pilot needs to be extracted by multiplying corresponding chaotic sequence, before channel estimation process. Performance difference in these two methods have been shown using simulation results.

This paper is organized as follows. In section II chaos based CDMA system with Bayesian estimator is shown. MAP and ML estimation algorithms are derived in section III. Simulation results are shown in section IV. Finally some concluding remarks are given in section V.

II. System model

Fig. 1 shows the baseband representation of the chaos based CDMA system with Bayesian channel estimator. In this figure channel estimation is performed after multiplying the chaotic signal to received signal. The wireless channel is assume to be quasi-static fading channel i.e. path gains are constant over a symbol duration. Then the received signal at user can be described as:

$$y(n) = \left(\sum_{k=1}^{N} \mathbf{s}_k^T(n) \mathbf{C}_k(n) \right) \mathbf{h}(n) + w(n) \tag{1}$$

where $\mathbf{s}(n) = [s(n), s(n-1), \cdots, s(n-L+1)]^T$ is the transmitted signal, $\mathbf{h}(n) = [h_0(n), h_1(n), \cdots, h_{L-1}(n)]^T$ is the quasi-static time varying channel for k^{th} user and $w(n)$ is the zero mean White Gaussian noise with variance of σ_w^2. L and N represents the total number of paths and users respectively. $\mathbf{C}(n) = diag[\mathbf{c}(n), \mathbf{c}(n-1), \cdots, \mathbf{c}(n-L+1)]$ is the diagonal matrix with elements $\mathbf{c}(\cdot)$ of length 2β known as

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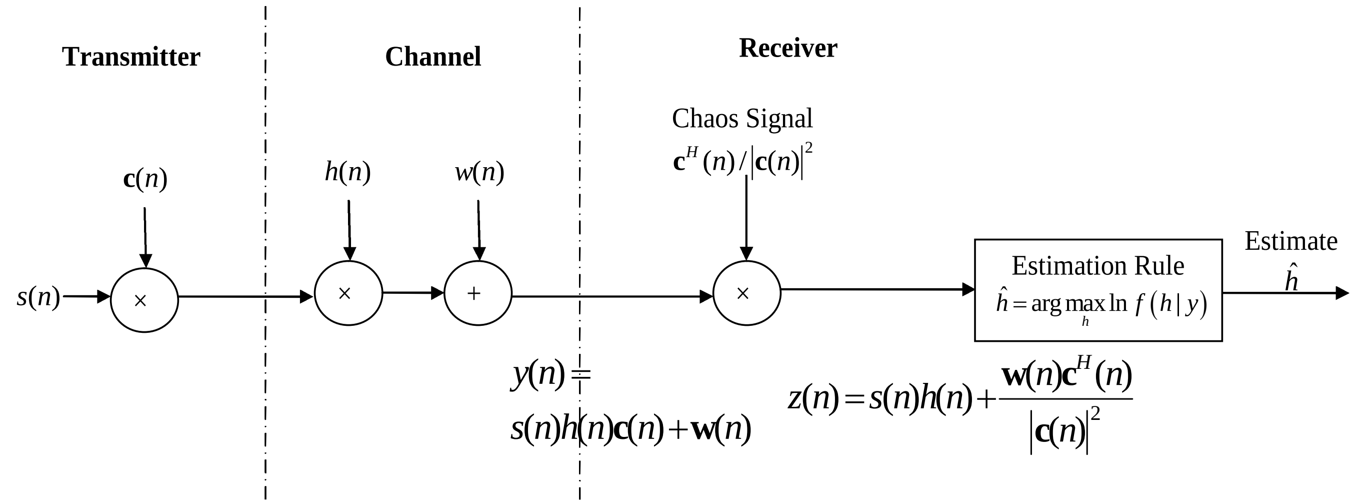

Fig. 1. Block Diagram of Proposed System

spreading factor. Subscript k denotes that the symbol is related to k^{th} user. Channel coefficients are assumed to be Gaussian distributed [27] i.e. $\mathbf{h} \sim N(\mathbf{m_h}, \sigma^2_{\mathbf{h}})$ where $\mathbf{m_h}$ and $\sigma^2_{\mathbf{h}}$ are the mean and variance of the channel respectively.

III. MAP and ML estimator

In this section we have derived Bayesian estimator equations for two cases i.e. for multiplexed pilot-data and add pilot-data case.

A. MAP and ML estimation with multiplexed pilot and user data

If pilot is multiplexed with user data then it can be extracted using demultiplexer at the receiver and can be processed by channel estimator. Here we have to assume that fading and nosie have same effect on pilot and user data symbols. In this case, the condition distribution function $p\left(y(n)|\mathbf{h}(n)\right)$ for k^{th} user is defined as

$$\begin{aligned} &p\left(y(n)|\mathbf{h}(n)\right) \\ &= \tfrac{1}{\sqrt{2\pi\sigma^2_w}} \exp\left(-\tfrac{\left(y(n)-\mathbf{s}^T_k(n)\mathbf{C}_k(n)\mathbf{h}(n)\right)^2}{2\sigma^2_w}\right) \end{aligned} \tag{2}$$

Since mean $\mathbf{m_h}$ and variance $\sigma^{\mathbf{2}}_{\mathbf{h}}$ of Gaussian distributed channel is known at receiver, therefore MAP estimation algorithm is given by

$$\begin{aligned} &\nabla_{\mathbf{h}} \left(-\tfrac{\left(y(n)-\mathbf{s}^T_k(n)\mathbf{C}_k(n)\mathbf{h}(n)\right)^2}{2\sigma^2_w}\right. \\ &\left. -\tfrac{(\mathbf{h}-\mathbf{m_h})\sigma^{\mathbf{-2}}_{\mathbf{h}}(\mathbf{h}-\mathbf{m_h})^T}{2} + \text{constants}\bigg|_{\mathbf{h}=\hat{\mathbf{h}}}\right) = 0 \end{aligned} \tag{3}$$

Above derivative reduces to following equation (see appendix A for derivation)

$$\begin{aligned} &\hat{\mathbf{h}}_{MAP}(n) = \\ &\mathbf{m_h} + \tfrac{1}{\sigma^2_w}\left(\sigma^{\mathbf{-2}}_{\mathbf{h}} + \tfrac{1}{\sigma^2_w}\mathbf{C}^T_k(n)\mathbf{s}_k(n)\mathbf{s}^T_k(n)\mathbf{C}_k(n)\right)^{-1} \\ &\times\mathbf{C}^T_k(n)\mathbf{s}_k\left(y(n) - \mathbf{s}^T_k(n)\mathbf{C}_k(n)\mathbf{m_h}\right) \end{aligned} \tag{4}$$

If we do not have a prior knowledge of the channel statistic, then we remove the second term in equation (3) and resultant algorithm is known as ML estimation i.e.

$$\nabla_{\mathbf{h}}\left(-\frac{\left(y(n)-\mathbf{s}^T_k(n)\mathbf{C}_k(n)\mathbf{h}(n)\right)^2}{2\sigma^2_w} + \text{constants}\bigg|_{\mathbf{h}=\hat{\mathbf{h}}}\right) = 0 \tag{5}$$

After solving derivative, we have following ML estimation equation

$$\hat{\mathbf{h}}_{ML}(n) = \left(\mathbf{s}^T_k(n)\mathbf{C}_k(n)\right)^{-1}y(n) \tag{6}$$

B. MAP and ML algorithm for added pilot and user data

For multiplexed pilot and user data, we have to assume same fading effects on both the signals. If we add pilot symbols to user symbols then fading have same effect on both the signals. Therefore same fading and noise effect assumptions can be removed. Further, in this case pilot has to be extracted from data for the channel estimation process. Since the chaotic sequences are orthogonal to each other therefore pilot symbols can be extracted by multiplying the received signal with chaotic sequence of pilot symbols. Multiplying received signal i.e. equation (1) with chaotic signal of k^{th} user we have

$$z(n) = y(n)\frac{\mathbf{C}^H_k(n)}{\mathbf{C}_k(n)\mathbf{C}^H_k(n)} \tag{7}$$

In this case MAP and ML estimation equations are given by (see appendix B for derivation)

$$\begin{aligned} &\hat{\mathbf{h}}_{MAP}(n) = \mathbf{m_h} + \tfrac{1}{\sigma^2_w}\left(\sigma^{\mathbf{-2}}_{\mathbf{h}} + \tfrac{1}{\sigma^2_w}\mathbf{s}_k(n)\mathbf{s}^T_k(n)\right)^{-1}\times \\ &\mathbf{s}_k\left(z(n) - \mathbf{s}^T_k(n)\mathbf{m_h}\right) \end{aligned} \tag{8}$$

and

$$\hat{\mathbf{h}}_{ML}(n) = \left(\mathbf{s}_k(n)\mathbf{s}^T_k(n)\right)^{-1}\mathbf{s}_k(n)z(n) \tag{9}$$

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IV. Simulation Results

In the simulation we compare the performance of estimators for three cases. In first case, the pilot is multiplexed with data without multiplying with chaotic sequences at the transmitter. We represent this case as 'Without Chaotic Multiplication' in the simulation results. Similarly other two cases i.e. multiplexed pilot-data and added pilot-data with chaotic sequences multiplication at transmitter, are denoted by 'Before Chaotic Multiplication' and 'After Chaotic Multiplication' respectively in the results.

The value of the spreading factor 2β is 50. Following Chebyshev polynomial function i is used to generate the chaotic sequence [32].

$$x_k = 1 - 2(x_{k-1})^2 \tag{10}$$

where x_k denotes the k^{th} chip value chaotic sequence.

Fig. 2 and Fig. 3 show the channel tracking performance of the three estimators at $0dB$ and $20dB$ SNR conditions respectively. Chaotic sequences spread the data over entire bandwidth during transmission and despreading takes place during reception. Further noise is spread by the chaotic sequence multiplication at the receiver. Due to this spreading of noise, performance of the estimator in the presence of chaotic sequence is better than the without chaotic spreading case, as shown in Fig. 2 and Fig. 3.

From these figures it is clear that performance of all the estimators are improved with increase in the SNR. Since chaotic sequences are directly used for channel estimation as well as noise spreading in multiplexed pilot-data case therefore its performance is better than the added pilot-data case, for lower SNR conditions. For higher SNR conditions performance of both the chaotic estimators are same as shown in Fig. 3.

Finally in Fig. 4, the BER performances are shown. $20dB$ performance improvement can be seen at $SNR = 10dB$ with chaotic spreading sequence over without spreading case. Further, performance improvement can be seen in 'Before Multiplication Case' over 'After Multiplication case' at lower SNR conditions.

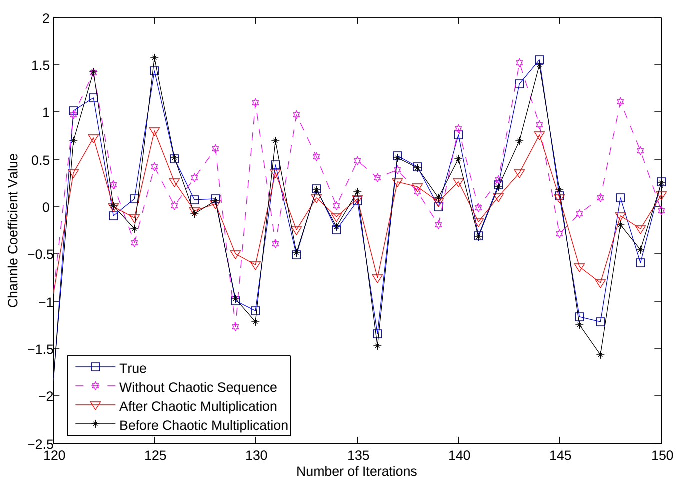

Fig. 2. MAP channel estimators performance, SNR = 0dB

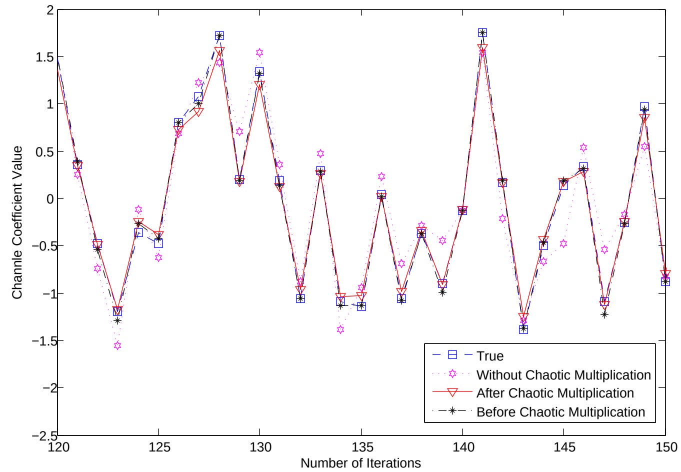

Fig. 3. MAP channel estimators performance, SNR = 20dB

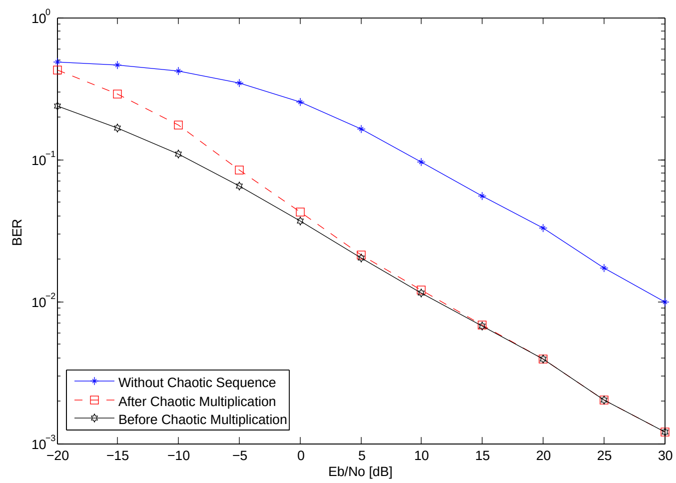

Fig. 4. BER performance of chaos based CDMA system using MAP, $2\beta = 50$

V. Conclusion

In this paper, we propose MAP channel estimation algorithms for chaos based CDMA system. In simulation, channel estimators are compared for various cases. In the first case, channel estimation is performed on the received signal directly i.e. in the presence of chaotic sequence. In second case chaotic sequence is not available for channel estimation because estimation is done after multiplying the received signal with chaotic sequence. The MAP estimators work better in the presence of chaotic sequence because in this case chaotic sequences are used for channel estimation as well as noise spreading. However, for this case pilot and data are multiplexed with each other, therefore we have to assume that pilot and data have same fading and noise effect. Further, if data and pilot have different fading effect then second estimation algorithm can be used.

Appendix A
Derivation of Equation (4)

Rewriting equation (3) after solving derivative, we have

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$$\hat{\mathbf{h}}(n) = \left(\sigma_{\mathbf{h}}^{-\mathbf{2}} + \frac{1}{\sigma_w^2}\mathbf{C}_k^T(n)\mathbf{s}_k(n)\mathbf{s}_k^T(n)\mathbf{C}_k(n)\right)^{-1} \times \left(\mathbf{m_h}\sigma_{\mathbf{h}}^{-\mathbf{2}} + \frac{\mathbf{C}_k^T(n)\mathbf{s}_k(n)y(n)}{\sigma_w^2}\right) \tag{11}$$

Let

$$\sigma_{\mathbf{h}}^{-\mathbf{2}} + \frac{1}{\sigma_w^2}\mathbf{C}_k^T(n)\mathbf{s}_k(n)\mathbf{s}_k^T(n)\mathbf{C}_k(n) = \mathbf{T} \tag{12}$$

Hence equation (11) becomes

$$\hat{\mathbf{h}}(n) = \mathbf{T}^{-1}\left(\mathbf{m_h}\sigma_{\mathbf{h}}^{-\mathbf{2}} + \frac{\mathbf{C}_k^T(n)\mathbf{s}_k(n)y(n)}{\sigma_w^2}\right) \tag{13}$$

Put the value of $\sigma_{\mathbf{h}}^{-\mathbf{2}}$ from equation (12) to (13), we get equation (4)

APPENDIX B
DERIVATION OF EQUATION (8) AND (9)

Putting the value of $y(n)$ from equation (1) in equation (7), we have

$$z(n) = \mathbf{s}_k^T(n)\mathbf{h}(n) + \left(\sum_{j=1,k\neq j}^{N}\mathbf{s}_k^T(n)\mathbf{C}_k(n)\right)\mathbf{h}(n)\times \frac{\mathbf{C}_k^T(n)}{\mathbf{C}_k(n)\mathbf{C}_k^T(n)} + \frac{w(n)\mathbf{C}_k^T(n)}{\mathbf{C}_k(n)\mathbf{C}_k^T(n)} \tag{14}$$

Since the cross-correlation of two different chaotic signals is very small, hence we can neglect the second term i.e.

$$z(n) \approx \mathbf{s}_k^T(n)\mathbf{h}(n) + \frac{w(n)\mathbf{C}_k^T(n)}{\mathbf{C}_k(n)\mathbf{C}_k^T(n)} \tag{15}$$

Now, following the same steps as in section III-A, we get equation (8) and equation (9).

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