The structural constant of an atom as the basis of some known physical constants

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***Abstract*—**Maxwell's equations are used in atomistic since it has been shown that they in addition to the emission of radiation include the simultaneous absorption and thereby maintain the stability of the atom. Conditions of existence of electromagnetic oscillation in the atom are met if the product of one part of the characteristic impedance of oscillator, called structural coefficient, and atomic number Z is constant. The structural constant of atom $s_0 \approx 8.278$ was determined through the stability of atoms and through the ionization energy of the hydrogen atom. The fine-structure constant is calculated from this, namely s_0 squared times two is inverse fine-structure constant 137.073. The meaning of the fine-structure constant is being discussed and proposes the use of these constants related to the isotope number. For this purpose, it is proposed to introduce a new unit. Planck's constant is obtained by multiplying the speed of light, magnetic constant, and charge of the electron and structural constant of atom squared. In a similar way, with the aid of structural constant of atom, Josephson's constant, von Klitzing's and Rydberg's constant were determined. These constants are here derived theoretically and are consistent with the measurements.

***Keywords*—**Fine-structure constant, Josephson constant, Planck constant, Structural constant of atom, von Klitzing constant.

I. Introduction

THE application of Maxwell's equations in the last hundred years has experienced a failure during the study phenomena in microcosm. Nevertheless, there are still tendencies of researchers to apply these equations except both, in the macro and micro world. The reasons for this tendency and for this paper as well include the fact that Maxwell's equations themselves do not possess a certain limit of their applicability. The main theoretical objection against the application of Maxwell's equations in the atomistic was that these equations require radiation of electrons, because there's acceleration of the charge on a circular orbit in an atom, causing the collapse of the atom. Since in reality the atoms do not collapse, the conclusion that Maxwell's equations are not valid in the atomistic seemed exactly right. One option has not been taken into account so far. It has been shown that the application of Maxwell's equations does not cause a collapse of atoms if these equations are observed completely, i.e., that in addition to simultaneous emission and absorption of radiation there [1], [2].Thus an atom with the application of Maxwell's equations remains stable. After that, there is no theoretical limit to the application of Maxwell's equations in the atom. Thus the atom can be treated as an electromechanical oscillator.

Using Maxwell's equations reveals the existence of a single constant of the atom that is made up of two structural parts, i.e., structural coefficient of Lecher's line $\sigma(\chi)$ and the atomic number Z. One of these parts is bonded to the structure of Lecher's line, which serves as a model of an electromagnetic wave in an atom, and the other one is related to the atomic number, which is part of an atom. So we called it *the structural constant of an atom* (shorter, *structural constant*) and we marked it with s_0 .

II. A Structural Constant and Its Calculation

In the Section IV, I will show the origin of a structural atom constant. Now let's say that it is defined as [3]

$$s_0 \equiv \sqrt{\sigma(\chi)\ Z}\,, \tag{1}$$

where s_0 is a structural atom constant, $\sigma(\chi)$ is a structural coefficient of Lecher's line [*Lecher line is twin-lead transmission line consisting of a pair of ideal conductive nonmagnetic wires of diameter* 2ρ, *separated by* δ, *situated in space with permittivity* ε *and permeability* μ, *whereby the argument* $\chi=\delta/\rho$, *and where the behavior of electromagnetic quantities as well as in the atom, in other words the Lecher line in our case represents the model of electromagnetic energy in the atom*], Z is an atomic number, which theoretically is not in integer domain. The capacitance and the inductance of Lecher's line per unit length are

$$C' = \frac{\pi\varepsilon}{\ln\left(\chi/2+\sqrt{\chi^2/4-1}\right)} \tag{2}$$

and

$$L' = \frac{\mu\left(\ln\chi+1/4\right)}{\pi}\,, \tag{3}$$

respectively. The characteristic impedance of Lecher's line is [4]

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$$Z_{LC} = \sqrt{\frac{L}{C}} = \sqrt{\frac{L'\Delta z}{C'\Delta z}} = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{\mu}{\varepsilon}}\,\frac{\sigma(\chi)}{\pi}. \qquad (4)$$

It follows by using (1), [3],

$$\sigma(\chi) = \sqrt{\left[\ln\left(\frac{\chi}{2} + \sqrt{\frac{\chi^2}{4} - 1}\right)\right]\left(\ln\chi + \frac{1}{4}\right)} = \frac{s_0^2}{Z}. \qquad (5)$$

Both guides in Lecher's line are set parallel in z-axis. The wave of current (i) and the wave of voltage (u), i.e., the current-voltage waves of Lecher's line, taking place in time t along the z-axis. These two mutually connected waves are described with two differential equations, [4]:

$$\frac{\partial^2 u}{\partial z^2} - L'C'\frac{\partial^2 u}{\partial t^2} = 0, \qquad \frac{\partial^2 i}{\partial z^2} - L'C'\frac{\partial^2 i}{\partial t^2} = 0. \qquad (6)$$

On the other hand the electromagnetic wave (with x-component of electric field strength E_x and y-component of magnetic field strength H_y) travels in general, but of course also within the atom, along the z-axis. This wave in the plane is also described with two differential equations, [4]:

$$\frac{\partial^2 E_x}{\partial z^2} - \varepsilon_0\mu_0\frac{\partial^2 E_x}{\partial t^2} = 0, \quad \frac{\partial^2 H_y}{\partial z^2} - \varepsilon_0\mu_0\frac{\partial^2 H_y}{\partial t^2} = 0. \quad (7)$$

From (6) and (7), we can see that the waves on the Lecher's line, and the electromagnetic waves in the atom, are described with the same form of differential equations. This means that all phenomena described by these equations are analogous. Therefore, it is possible to adapt the equations of current and voltage waves on Lecher's line, with the help of one factor F, so that it describes an electromagnetic wave in the atom, *e.g.*, from (6) and (7) we obtain third equation, in this way,

$$\begin{gathered}\frac{\partial^2 u}{\partial z^2} - L'C'\frac{\partial^2 u}{\partial t^2} = 0, \qquad \frac{\partial^2 E_x}{\partial z^2} - \varepsilon_0\mu_0\frac{\partial^2 E_x}{\partial t^2} = 0,\\ \frac{\partial^2 u}{\partial z^2} - F^2 L'C'\frac{\partial^2 u}{\partial t^2} = 0,\end{gathered} \qquad (8)$$

which means that by multiplying $L'C'$ by a factor F^2 wave on Lecher's line behaves according to the phase velocity like electromagnetic wave in an atom. Therefore worth [as follows from (8)],

$$F^2 L'C' = \varepsilon_0\mu_0, \qquad (9)$$

or

$$F = \sqrt{\frac{\varepsilon_0\mu_0}{\varepsilon\mu}\,\frac{\ln\left(\chi/2 + \sqrt{\chi^2/4 - 1}\right)}{\ln\chi + 1/4}}, \qquad (10)$$

as gives in the case of Lecher's line within the vacuum ($\varepsilon = \varepsilon_0$, $\mu = \mu_0$):

$$F(\chi) = \sqrt{\frac{\ln\left(\chi/2 + \sqrt{\chi^2/4 - 1}\right)}{\ln\chi + 1/4}}. \qquad (11)$$

This further means that the phase velocity of the electromagnetic wave in an atom, u_{em}, can be expressed as

$$\begin{aligned}\frac{1}{u_{em}^2} &= \left[F(\chi)\right]^2 L'C' = \varepsilon_0\mu_0 = \frac{1}{c^2},\\ u_{em} &= \frac{1}{\sqrt{\varepsilon_0\mu_0}} = \lambda_{em}\nu_{em} = c\end{aligned} \qquad (12)$$

while the phase velocity of the wave of current and voltage at the Lecher's line, u_{CV}, using (2), (3) and (11), can be expressed as (with $\varepsilon = \varepsilon_0$, $\mu = \mu_0$, which is an assumption we will continue to count with)

$$\frac{1}{u_{CV}^2} = L'C' = \frac{\varepsilon\mu\left(\ln\chi + 1/4\right)}{\ln\left(\chi/2 + \sqrt{\chi^2/4 - 1}\right)}, \qquad (13)$$

$$\begin{aligned}u_{CV} &= \sqrt{\frac{\ln\left(\chi/2 + \sqrt{\chi^2/4 - 1}\right)}{\varepsilon_0\mu_0\left(\ln\chi + 1/4\right)}}\\ &= \lambda_{CV}\nu_{CV} = c\,F(\chi)\ ,\end{aligned} \qquad (14)$$

where $c = 1/\sqrt{\varepsilon_0\mu_0}$ is the speed of light in vacuum, λ_{CV} is the wavelength of the wave of current and voltage on the Lecher's line, and the ν_{CV} is the frequency of the same wave. Each electromagnetic wave in the atom thus is associated with one wave of current and voltage of Lecher's line. With that, following the behavior of waves on Lecher's lines we follow the behavior of the waves in the atom. Thus we connect the phase relationships of the waves on the Lecher's line and electromagnetic wave in the atom.

Equation (5) connects each specific atomic number Z to a specific argument χ, for which the wave of current and voltage on Lecher's line is analogous with phase velocity of the electromagnetic wave in that atom [5]. In theoretical considerations Z is treated here as a continuous physical quantity which in reality takes discrete integer values. Later

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we will connect electromagnetic energy of Lecher's line with the energy of the electromagnetic wave in the atom.

There may be at least three methods of determining the structural constant s_0 .

- The first method is linked with the just treated phase velocity of the electromagnetic wave in the atom and the stability of atoms, where we need only one measured data (the atomic number Z of the first unstable atoms) [4]. In other words, this way answers the question, what is structural constant of atoms if the first unstable atom has atomic number Z. The percentage uncertainty of this method is 0.3%.
- The second method is connected to the ionization energy of hydrogen, where we need four data (ionization voltage V, electron charge e, electron mass m and the speed of light in vacuum c). The relative standard uncertainty is 1.3×10^{-8}. It is three times better than that of Planck's constant, where the relative standard uncertainty, according to the NIST, is 4.4×10^{-8}.
- The third method is an extension of the second. Instead of the energies we use frequencies or wavelengths of spectral lines of atoms. Here we need six data (wavelength of the radiation of atoms λ, electron charge e, electron mass m, speed of light in vacuum c, magnetic constant μ_0 and the initial velocity of electrons β_0). The relative standard uncertainty is therefore less favorable than in the second case, so we will not implement this method here.

III. Calculation of a Structural Constant Through the Stability of Atoms

The first method for the calculation of s_0 is based on analysis of the stability of atoms [6]. It was concluded that the phase velocity u_{CV} on Lecher's line decreases towards zero when the argument $\chi = \chi_0 = 2.328\,788$ or less, [4], Fig. 1. In this case the second derivative (in absolute value) becomes greater than 1, indicating a sharp drop the phase velocity, which leads to instability of the waves on the Lecher's line, that of the associated electromagnetic wave in the atom, and thus to instability of the whole atom.

The argument χ_0 therefore corresponds to the first unstable atom, and in 2003 it was found that it is bismuth, a chemical element with symbol Bi and atomic number 83, i.e., ^{83}Bi. The beginning of the instability series of the atoms may belong to the lower or upper part of the number 83, i.e., Z=83±1/2. Using the above Z and $\sigma(\chi_0)$ = σ(2.382788) = 0.825402, we get from (1) the result s_0 = 8.277 ± 0.025, which means that percentage uncertainty is 0.3%. The validity of this result will be confirmed later by using NIST Atomic Spectra Database.

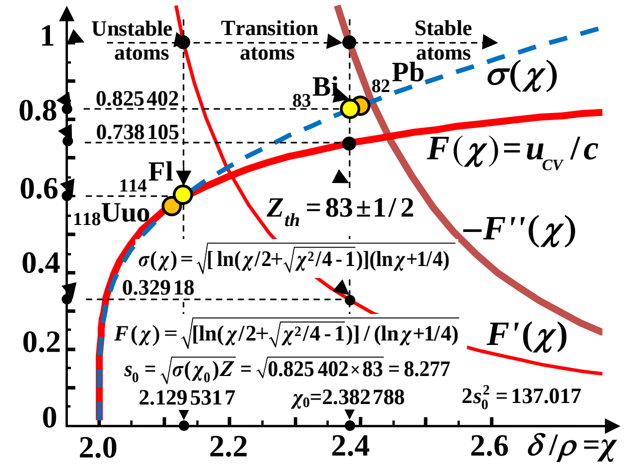

Fig. 1 Structural coefficient of Lecher's line $\sigma(\chi)$, normalized phase velocity of current and voltage of Lecher's line $F(\chi)=u_{CV}/c$, the first derivative of the normalized phase velocity $F'(\chi)$, inverted second derivative of the normalized phase velocity $F''(\chi)$, all versus ratio $\delta/\rho=\chi$ of the transmission Lecher's line, consisting of a pair of ideal conducting nonmagnetic parallel wires of radius ρ separated by δ**.**

IV. Calculation of a Structural Constant Through the Ionization Energy of the Hydrogen

In similar manners as a phase velocities we also associate electromagnetic energy of Lecher's line and the energy of the electromagnetic wave in the atom. In this sense, we require that energy of an electromagnetic wave in an atom is equal to the electromagnetic energy of Lecher's line.

The electromagnetic energy in the atom is obtained by using the energy balance. Newton's second law and Coulomb's law together give [7]

$$\frac{m\,v^2}{r\sqrt{1-\beta^2}} = \frac{|qQ|}{4\pi\varepsilon_0\, r^2}, \tag{15}$$

which gives

$$r = \frac{|qQ|}{4\pi\varepsilon_0\, mc^2}\,\frac{\sqrt{1-\beta^2}}{\beta^2}, \tag{16}$$

where r stands for the radius of the circular orbit of the electron, q is the charge of the electron (q=-e), e is elementary charge, Q is the charge of the nucleus (Q=Ze), m is the electron rest mass, v is the velocity of the electron, $\beta = v/c$, ε_0 is permittivity of free space, μ_0 is permeability of free space, $m\big/\sqrt{1-\beta^2}$ is the transverse mass of the electron [8]. Increasing transverse mass of the electron is $\Delta m = m\big/\sqrt{1-\beta^2} - m$. The kinetic energy of an electrons, [7],

$$K = \Delta m\, c^2 = \frac{mc^2}{\sqrt{1-\beta^2}} - mc^2, \tag{17}$$

and its potential energy [7], using (16),

$$U = \frac{qQ}{4\pi\varepsilon_0 r} = -\frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{mc^2}{\sqrt{1-\beta^2}}\beta^2 . \qquad (18)$$

A point charge Q created at a distance r from the charge (relative to the potential at infinity) the electric potential Φ:

$$\Phi = \frac{Q}{4\pi\varepsilon_0 r}, \qquad (19)$$

so that the potential energy according to (18) can be written

$$U = q\Phi . \qquad (20)$$

If we know the amount of potential energy U then we can determine β from (18):

$$\beta = \frac{1}{\sqrt{2}\, mc^2}\sqrt{\sqrt{U^4 + \left(2mc^2U\right)^2} - U^2} . \qquad (21)$$

The total mechanical energy of an electron (W) is the sum of its kinetic and potential energy [7]:

$$W = K + U = -mc^2\left(1-\sqrt{1-\beta^2}\right) . \qquad (22)$$

If we know the total mechanical energy $W = -eV$, then from (21) we can also determine β:

$$\beta = \sqrt{1-\left(1-\frac{eV}{mc^2}\right)^2}, \qquad (23)$$

where V is the potential difference through which the electron passes to get an equal energy as electromagnetic energy E_{em} or ΔE_{em}. This energy corresponds to the energy of a photon.

According to the law of conservation of energy the energy of an isolated system, which includes the total energy of electrons W and electromagnetic energy of the atom E_{em}, is constant, despite internal changes (the energy disappearing in one form and reappearing in another or is transferred from one object to another):

$$W + E_{em} = const., \qquad (24)$$

i.e.,

$$\Delta W + \Delta E_{em} = 0, \qquad (25)$$

and using (22) we get

$$\begin{aligned}\Delta W &= \int_{\beta_0}^{\beta_n} dW = \int_{\beta_0}^{\beta_n}\left(-\frac{mc^2\beta}{\sqrt{1-\beta^2}}\right)d\beta \\ &= -mc^2\left(\sqrt{1-\beta_0^2}-\sqrt{1-\beta_n^2}\right) \\ &= -eV = -\Delta E_{em},\end{aligned} \qquad (26)$$

where β_0 is the initial velocity and β (or β_n) is a final velocity of the electron ($\beta_0 \le \beta_n$). From (26) we get

$$\sqrt{1-\beta_n^2} = \sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{mc^2} \qquad (27)$$

or

$$\begin{aligned}\beta_n^2 &= 1-\left(\sqrt{1-\beta_0^2}-\frac{\Delta E_{em}}{mc^2}\right)^2 \\ &= \frac{2\Delta E_{em}}{mc^2}\left(\sqrt{1-\beta_0^2}-\frac{\Delta E_{em}}{2mc^2}+\frac{mc^2\beta_0^2}{2\Delta E_{em}}\right),\end{aligned} \qquad (28)$$

which means it is

$$\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{2mc^2} + \frac{mc^2\beta_0^2}{2\Delta E_{em}} = \frac{mc^2\beta_n^2}{2\Delta E_{em}} . \qquad (29)$$

Using (27) and (28), (16) becomes

$$r = \frac{\frac{1}{2}\frac{|qQ|}{4\pi\varepsilon_0\Delta E_{em}}\left(\sqrt{1-\beta_0^2}-\frac{\Delta E_{em}}{mc^2}\right)}{\sqrt{1-\beta_0^2}-\frac{\Delta E_{em}}{2mc^2}+\frac{mc^2\beta_0^2}{2\Delta E_{em}}}, \qquad (30)$$

or using (18), (29) and (30)

$$\Delta E_{em} = \frac{\frac{1}{2}\frac{|qQ|}{4\pi\varepsilon_0 r}\left(\sqrt{1-\beta_0^2}-\frac{\Delta E_{em}}{mc^2}\right)}{\sqrt{1-\beta_0^2}-\frac{\Delta E_{em}}{2mc^2}+\frac{mc^2\beta_0^2}{2\Delta E_{em}}} \qquad (31)$$

and from (29) and (31) we get

$$|U| = \frac{mc^2\beta_n^2}{\sqrt{1-\beta_0^2}-\frac{\Delta E_{em}}{mc^2}} . \qquad (32)$$

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In addition to equalizing the phase velocity of the waves in the atom and the waves on the Lecher's line, will also equalize the energy ΔE_{em} of these waves. Let's find now an expression for wave energy on the Lecher's line. Lecher's line can be represented as an inductive-capacitive network (so-called *LC* network), which finally makes the oscillatory circuit (*LC* circuit) [9]. The natural frequency ν_{LC} of *LC* circuit is [7]

$$\nu_{LC} = \frac{1}{2\pi\sqrt{LC}}, \tag{33}$$

where *C* is the sum of all small capacitances of the *LC* network on the open end of the network, and *L* is the sum of all small inductances of the *LC* network on the short-circuited of the network [9]. This frequency is equal to the frequency ν of the electromagnetic wave generated in the atom

$$\nu = \nu_{LC}. \tag{34}$$

Electromagnetic energy in the *LC* circuit E_{LC} is equal to the maximum amount of energy on the inductor with an inductance *L*, or energy on the capacitor with a capacitance *C* [7],

$$E_{LC} = \frac{1}{2}\frac{\Theta^2}{C} = \Delta E_{em}, \tag{35}$$

where Θ stands for maximal charge on the mentioned capacitor with the capacitance *C*. Using (31) and (35) we obtain

$$\frac{1}{2}\frac{\Theta^2}{C} = \frac{\frac{1}{2}\frac{|qQ|}{4\pi\varepsilon_0 r}\left(\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{mc^2}\right)}{\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{2mc^2} + \frac{mc^2\beta_0^2}{2\Delta E_{em}}}. \tag{36}$$

One equation (36) has two unknowns, Θ and *C*. By using Diophantine equations we get next couple of the many solutions [10]:

$$C = 4\pi\varepsilon_0 r, \tag{37}$$

$$\Theta^2 = \frac{|qQ|\left[\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{mc^2}\right]}{\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{2mc^2} + \frac{mc^2\beta_0^2}{2\Delta E_{em}}}. \tag{38}$$

Equation (35), which represents the energy of the oscillatory *LC* circle, or the energy of the electromagnetic wave in the atom, can be written by using (33), (34) and (38):

$$\begin{aligned}\frac{1}{2}\frac{\Theta^2}{C} &= \frac{1}{2}\frac{\pi}{\pi}\frac{\Theta^2}{\sqrt{C}\sqrt{C}}\frac{\sqrt{L}}{\sqrt{L}} \\ &= \pi\sqrt{\frac{L}{C}}\Theta^2\frac{1}{2\pi\sqrt{LC}} \\ &= \pi Z_{LC}\Theta^2\nu = A\nu \end{aligned} \tag{39}$$

$$= \Delta E_{em} = \frac{\pi Z_{LC}|qQ|\left(\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{mc^2}\right)}{\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{2mc^2} + \frac{mc^2\beta_0^2}{2\Delta E_{em}}}\nu,$$

where

$$A = \frac{\pi Z_{LC}|qQ|\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{mc^2}}{\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{2mc^2} + \frac{mc^2\beta_0^2}{2\Delta E_{em}}} \tag{40}$$

is the action of electromagnetic oscillator. One part of it which does not depend on β_0 or on ΔE_{em} can be denoted by A_0:

$$A_0 = \pi Z_{LC}|qQ|, \tag{41}$$

thus applies

$$A = A_0\frac{\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{mc^2}}{\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{2mc^2} + \frac{mc^2\beta_0^2}{2\Delta E_{em}}}. \tag{42}$$

From (4) (i.e., $Z_{LC} = \sqrt{L/C}$), (32) and (33) [i.e., $\nu_{LC} = \nu = 1/(2\pi\sqrt{LC})$] we obtain $\nu = 1/(2\pi Z_{LC}C)$. Using (37) and the last part of (4) (provided that $\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $c = 1/\sqrt{\varepsilon_0\mu_0}$) we get:

$$\nu = \frac{c}{8\pi\sigma(\chi)r}. \tag{43}$$

As we can see from (43) frequency ν depends on the speed of light c and two spatial parameters, r and $\sigma(\chi)$. Note that this frequency is not in any way dependent on the charges. Inserting *r* from (30) into (43) gives:

$$\nu = \frac{\Delta E_{em}}{\mu_0 c\sigma(\chi)|qQ|}\frac{\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{2mc^2} + \frac{\beta_0^2 mc^2}{2\Delta E_{em}}}{\sqrt{1-\beta_0^2} - \frac{\Delta E_{em}}{mc^2}}. \tag{44}$$

The equation (44) is dependent now on the charge $|qQ|$. Such a dependence of the charge, in accordance with (43), therefore not exists. The physical quantity which has the dimension of the charge may be in expression (44), but its amount cannot be changed. Thus, (44) should be made independent of the charge. Such "neutralizing" of this equation can be achieved by converting the product $\sigma(\chi)|qQ|$ in a constant, let's call it *charge constant of the structure*, q_0, in the following way:

$$\sigma(\chi)|qQ| = q_0^2. \tag{45}$$

Because

$$|qQ| = Ze^2, \tag{46}$$

applies

$$\sigma(\chi)Z = \left(\frac{q_0}{e}\right)^2 = s_0^2. \tag{47}$$

Thus we derived equation (1), i.e., $s_0 = \sqrt{\sigma(\chi)\,Z}$, as promised at the beginning. From (4), (41) and (47) follows:

$$\begin{aligned} A_0 &= \pi Z_{LC}\,|qQ| = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sigma(\chi)Ze^2 \\ &= \sqrt{\frac{\mu_0}{\varepsilon_0}}s_0^2e^2 = c\mu_0 s_0^2 e^2. \end{aligned} \tag{48}$$

Equation (44), using (48), we can write:

$$\Delta E_{em} = A_0\nu\frac{\sqrt{1-\beta_0^2} - \dfrac{\Delta E_{em}}{mc^2}}{\sqrt{1-\beta_0^2} - \dfrac{\Delta E_{em}}{2mc^2} + \dfrac{mc^2\beta_0^2}{2\Delta E_{em}}}. \tag{49}$$

Combining (29) and (49) yields

$$\Delta E_{em} = mc^2\left(\sqrt{1-\beta_0^2} - \frac{1}{2}\frac{mc^2\beta_n^2}{A_0\nu}\right). \tag{50}$$

Arranging and solving of (49) leads to

$$\begin{aligned} \Delta E_{em} &= eV \\ &= A_0\nu + mc^2\sqrt{1-\beta_0^2} - \sqrt{\left(A_0\nu\right)^2 + \left(mc^2\right)^2}. \end{aligned} \tag{51}$$

From (35), (39) and (50) follows

$$\Delta E_{em} = A\nu, \tag{52}$$

and thence

$$A = A_0 + \frac{mc^2\sqrt{1-\beta_0^2}}{\nu} - \sqrt{A_0^2 + \left(\frac{mc^2}{\nu}\right)^2}. \tag{53}$$

By organizing (44), using (45), (46) and (47), we obtain:

$$\nu = \frac{\Delta E_{em}}{A_0}\frac{\sqrt{1-\beta_0^2} - \dfrac{\Delta E_{em}}{2mc^2} + \dfrac{mc^2\beta_0^2}{2\Delta E_{em}}}{\sqrt{1-\beta_0^2} - \dfrac{\Delta E_{em}}{mc^2}}, \tag{54}$$

which can be written in the form of extended Duane-Hunt's law,

$$\nu = \frac{eV}{A_0}\frac{\sqrt{1-\beta_0^2} - \dfrac{eV}{2mc^2} + \dfrac{mc^2\beta_0^2}{2eV}}{\sqrt{1-\beta_0^2} - \dfrac{eV}{mc^2}}, \tag{55}$$

or with use of (40) and (41)

$$\nu = \frac{eV}{A}. \tag{56}$$

From (27), (28) and (55) follows

$$A_0\nu = \frac{1}{2}\frac{mc^2\beta^2}{\sqrt{1-\beta^2}} = \frac{1}{2}\frac{m}{\sqrt{1-\beta^2}}\upsilon^2, \tag{57}$$

which according to (18) gives

$$\nu = \frac{|U|}{2A_0}. \tag{58}$$

The solution of (57) gives β if we know A_0, m, c and ν:

$$\beta = \frac{\sqrt{2}}{mc^2}\sqrt{\sqrt{\left(A_0\nu\right)^4 + \left(mc^2A_0\nu\right)^2} - \left(A_0\nu\right)^2}, \tag{59}$$

which together with (58) is the same as (21).

The momentum of the electromagnetic wave in an atom p_{em} is defined as the momentum of photon [7], [8], which means that it is equal to the ratio of energy ΔE_{em} and the phase velocity u_{em} of electromagnetic wave in the atom, i.e.,

$$u_{em} = \lambda\nu, \tag{60}$$

whereby λ is wavelength of the electromagnetic wave in an atom. So momentum of electromagnetic wave in the atom reads

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$$p_{em} = \frac{\Delta E_{em}}{u_{em}} = \frac{\Delta E_{em}}{\lambda \nu}, \qquad (61)$$

and using (38) this expression becomes

$$p_{em} = \frac{A\nu}{\lambda \nu} = \frac{A}{\lambda}. \qquad (62)$$

In accordance with the law of conservation of momentum, the momentum of the electromagnetic wave p_{em} is equal to the relativistic momentum of the electron [7], $p = mv / \sqrt{1-v^2/c^2}$,

$$\frac{A}{\lambda} = \frac{m\,v}{\sqrt{1-\beta^2}} = \frac{mc\beta}{\sqrt{1-\beta^2}}. \qquad (63)$$

With the use of (27), (28), (40) and (41) from (63) we get (with $\Delta E_{em} = eV$):

$$\lambda = \frac{A}{mv}\sqrt{1-\beta^2} = \frac{A}{mc}\frac{\sqrt{1-\beta^2}}{\beta} = \frac{A_0}{\sqrt{2meV}} \frac{\left(\sqrt{1-\beta_0^2} - \frac{eV}{mc^2}\right)^2}{\sqrt{\sqrt{1-\beta_0^2} - \frac{eV}{2mc^2} + \frac{mc^2\beta_0^2}{2eV}}^{\,3}}. \qquad (64)$$

The phase velocity of electromagnetic wave in an atom, according to (55), (60) and (64):

$$u_{em} = \sqrt{\frac{eV}{2m}} \frac{\sqrt{1-\beta_0^2} - \frac{eV}{mc^2}}{\sqrt{\sqrt{1-\beta_0^2} - \frac{eV}{2mc^2} + \frac{mc^2\beta_0^2}{2eV}}}. \qquad (65)$$

From (42), (62) and (64) follows

$$p_{em} = \sqrt{2meV} \frac{\sqrt{\sqrt{1-\beta_0^2} - \frac{eV}{2mc^2} + \frac{mc^2\beta_0^2}{2eV}}}{\sqrt{1-\beta_0^2} - \frac{eV}{mc^2}}. \qquad (66)$$

Using (30), (64) and (65) we obtain

$$\frac{\lambda}{r} = \frac{8\pi\varepsilon_0 A_0}{|qQ|}\sqrt{\frac{eV}{2m}} \frac{\sqrt{1-\beta_0^2} - \frac{eV}{mc^2}}{\sqrt{\sqrt{1-\beta_0^2} - \frac{eV}{2mc^2} + \frac{mc^2\beta_0^2}{2eV}}} = \frac{8\pi\varepsilon_0 A_0}{|qQ|} u_{em}. \qquad (67)$$

From (18) and (67) follows

$$\lambda = 2A_0 \frac{u_{em}}{|U|}. \qquad (68)$$

On the other hand, the same equation (68) we also obtain using (58) and (60).

At least two separate oscillatory processes are simultaneously performed within atoms. It is on the one hand uniform circular motion of electrons around the nucleus, as well as with other the oscillation electromagnetic energy generated within atoms, which acts as an electromagnetic wave in an atom [1]. The time for one complete revolution of electrons around the nucleus (the period T) is

$$T = \frac{2r\pi}{v} = \frac{1}{f}, \qquad (69)$$

f is the frequency of rotation. The period of electromagnetic wave, T_{em}, is

$$T_{em} = \frac{1}{\nu}. \qquad (70)$$

The multiplication of (69) with ν gives

$$\frac{\nu}{f} = \frac{2r\pi\nu}{v}. \qquad (71)$$

Using (60) and (68) from (71) we obtain

$$\frac{\nu}{f} = \frac{|qQ|}{4\varepsilon_0 A_0 v}. \qquad (72)$$

The electromagnetic wave in an atom can exist as a standing wave [11]. Standing wave does not transmit the energy, but it sways existing energy. If the frequency of the standing wave is ν, active power (index *ap*) of the standing wave oscillates with dual frequency $f_{ap} = 2\nu$. To electromagnetic standing wave took place in an atom there must be a mutual harmony between two above-mentioned processes. It means that the frequency f_{ap} of active power must be an integer relationship with the frequency of rotation f ,

$$f_{ap} = n\ f\ , \tag{73}$$

where n is one of the hole numbers 1, 2, 3, … Both above-mentioned processes in respect of synchronization are equal, so also applies

$$f = n\ f_{ap}\ . \tag{74}$$

Two equations, (73) and (74), can be written in the form of only one expression,

$$f_{ap} = n^{\pm 1} f\ , \tag{75}$$

or

$$2\nu = n^{\pm 1} f \tag{76}$$

From (72) and (76) we obtain the velocity of electrons υ_n:

$$\upsilon_n = \frac{1}{n^{\pm 1}} \frac{|qQ|}{2\varepsilon_0 A_0}\ . \tag{77}$$

From (76) follows:

$$\nu_n = \frac{1}{2} n^{\pm 1} f_n\ . \tag{78}$$

From (48) and (77) follows

$$s_0 = \sqrt{\frac{Z}{2n^{\pm 1}\beta_n}}\ , \tag{79}$$

and from (28)

$$\beta_n = \sqrt{1 - \left(\sqrt{1-\beta_0^2} - \frac{eV}{mc^2} \right)^2}\ . \tag{80}$$

From (79) and (80) follows

$$s_0 = \sqrt{\frac{Z}{2n^{\pm 1}\sqrt{1 - \left(\sqrt{1-\beta_0^2} - \frac{eV}{mc^2} \right)^2}}} \tag{81}$$

The equation (81) is a constant, independent of the variables β_0, n, V and Z. For an accurate calculation of the constant s_0 we need only one precise measurement. It can be made through data ionization of hydrogen atoms. In this case in (81) is: Z=1, β_0=0, $n^{\pm 1}$=1

$$s_0 = \sqrt{\frac{1}{2\sqrt{1 - \left(1 - \frac{eV}{mc^2} \right)^2}}}\ , \tag{82}$$

with NIST data, http://www.nist.gov/pml/data/asd.cfm, eV=13.598 434 005 136(12) eV and CODATA 2010 recommended values of fundamental physical constants, e = 1.602 176 565(35) × 10^{-19} C, m = 9.109 382 91(40) × 10^{-31} kg, c=299 792 458 m/s, we get $s_0 = 8.278\ 691\ 78(11)$, with relative standard uncertainty 1.3×10^{-8}. This is in accordance with the results obtained in the section III.

V. The Fine-Structure Constant and the Structural Constant of Atom

From (79) we get

$$\beta_n = \frac{1}{n^{\pm 1}} \frac{Z}{2s_0^2}\ , \tag{83}$$

where $n^{\pm 1}$ stands for n^{+1}=1,2,3, … or n^{-1}=1,2,3, …, depending on whether the orbits away or closer to the atomic nucleus, respectively.

The maximum amount of $\beta = \upsilon / c$ is 1. This is the case when Z is a maximum at the same time when $n^{\pm 1}$ is a minimum (a minimum of n^{+1} and n^{-1} is one, i.e., $n^{\pm 1}$=1). Therefore, the maximum atomic number theoretical is $2s_0^2$, and really the maximum atomic number Z_{max} is an integer of $2s_0^2$, i.e.,

$$Z_{max} = \text{Integer}\left(2s_0^2\right)\ . \tag{84}$$

We'll connect now three physical quantities, the number of different elements, isotope number and fine-structure constant. The number of different elements is the maximum atomic number $2s_0^2$. If we want to describe some isotope (now existing 3 179), it is necessary to define a physical quantity, let's call it '*isotope number*' and determine its *unit*. The unit of isotope number can be fraction $1/(\mathit{fine-structure\ constant})$ of the maximum atomic number. My proposal is that it's named *boskovic*, in honor Croatian scientists in the 18th century, Roger Joseph Boscovich (Rudjer Josip Boskovic), which dealt with atomistic and described it in its work in Latin *Theoria Philosophiæ Naturalis* from 1763. The symbol of this unit can be B.

Then we would have 8 basic units: 1 - meter (m), unit of length, 2 - kilogram (kg), mass units, 3 - second (s), unit of time, 4 - ampere (A), unit of electrical current, 5 - kelvin (K), unit of thermodynamic temperature, 6 - candela (cd), unit of luminous intensity, 7 - mole (mol), unit of amount of

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substance, and additionally, 8 - boskovic [B], unit of isotope number, Table I.

We can say now that the f*ine-structure constant* is a unit of *number of different elements*. It is the fraction $1/137.073\,475\,176\,479\,14$ of the *maximum number of different elements*, i.e.,

$$\alpha = \frac{1}{2s_0^2}. \tag{85}$$

Comparing (82) and (85) we get $\alpha = \sqrt{1-\left(1-eV/mc^2\right)^2}$.

Thus, $S = 1\frac{0}{7}\left[\mathrm{B}\right]$ represents the hydrogen ${}_1^1\mathrm{H}$, $S = 1\frac{1}{7}\left[\mathrm{B}\right]$ represents the hydrogen ${}_1^2\mathrm{H}$, $S = 1\frac{2}{7}\left[\mathrm{B}\right]$ represents the hydrogen ${}_1^3\mathrm{H}$, … $S = 1\frac{6}{7}\left[\mathrm{B}\right]$ represents last isotope of hydrogen ${}_1^7\mathrm{H}$. Generally, when an atom has *Z* protons and *D* isotopes, then for his *i*-th isotope worth (Fig. 2 and Table I).

$$S = \left(Z + \frac{i\text{-}1}{D}\right)\left[\mathrm{B}\right]. \tag{86}$$

It should be noted that the possible and different formula, say those who would rather *D* had assumed the maximum possible number of isotopes of an element, or similar.

VI. The Planck's Constant and the Structural Constant of Atom

The equations (52) and (53) describe the photon energy $\Delta E_{em} = A\nu$. According to Einstein's proposal this energy is equal to $h\nu$, where *h* is Planck's constant and ν is the frequency of electromagnetic wave (i.e., the light). This means that $h = A$ and in accordance with (53)

$$h = A = A_0 + \frac{mc^2\sqrt{1-\beta_0^2}}{\nu} - \sqrt{A_0^2 + \left(\frac{mc^2}{\nu}\right)^2}. \tag{87}$$

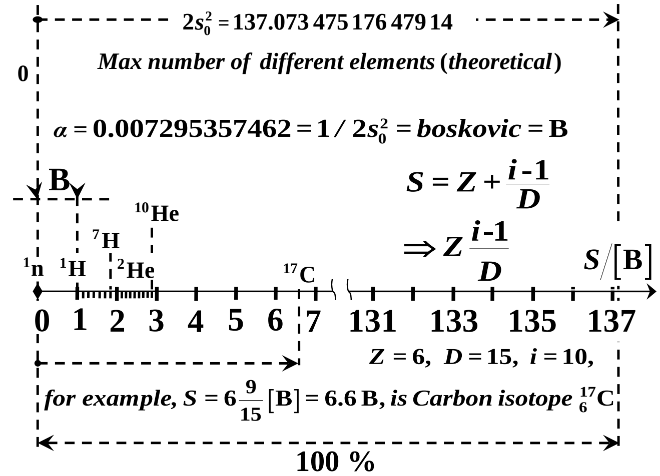

Fig. 2 The fine-structure constant α presented as a fraction 1/137.073 475 176 479 14 of the maximum number of different elements, and the proposed introduction of unit isotope number named *boskovic* with the symbol B. In that way one can assign a number *S* to each of the isotope (now there are more than 3 179 isotopes)

It is obvious that Planck's *h* depends on the frequency ν. For low frequencies applies.

$$h \approx A_0 = c\,\mu_0 e^2 s_0^2. \tag{88}$$

It is more correct to speak of the constants A_0 and the energy of photons

$$\Delta E_{em} = A_0\nu + mc^2\sqrt{1-\beta_0^2} - \sqrt{\left(A_0\nu\right)^2 + \left(mc^2\right)^2}, \tag{89}$$

but the Planck's *h* and the energy of the photons $\Delta E_{em} = h\nu$.

Table I Proposal: the basic units with the addition of unit of isotope number called *boskovic* [B]. The boskovic, unit of isotope number, is the fraction 1/137.073 475 176 479 of the maximum number of different elements

Quantity	Sign	Unit	Symbol
Length	*l*	meter	m
Mass	*m*	kilogram	kg
Time	*t*	second	s
Electrical current	*I*	ampere	A
Thermodynamic temperature	*T*	kelvin	K
Luminous intensity	*I*	candela	cd
Amount of substance	*n*	mole	mol
Isotope number	*S*	boskovic	B

Table II An abbreviated list of the fundamental constants of physics and chemistry which are made using structural constant of atoms s_0, electron mass m, proton mass μ_P, magnetic constant μ_0 and speed of light in vacuum c

Quantity	Symbol	Equation	Numerical value	Unit	% Difference*
structural constant of atom	s_0	$\sqrt{\sigma(\chi)Z}$	$8.278\ 691\ 78(11)$		not known
inverse fine-structure const.	α^{-1}	$2s_0^2$	$137.073\,475\,176\,479$		+0.027
fine-structure constant	α	$1/2s_0^2$	$7.295\,357\,462\,212\times10^{-3}$		-0.027
von Klitzing constant	R_K	$\mu_0 c s_0^2$	$2.581\,280\,744\,340\times10^{4}$	Ω	+0.027
action constant, Planck's h	A_0	$\mu_0 c s_0^2 e^2$	$6.627\,882\,090\,554\times10^{-34}$	J s	+0.027
conversion constant	K_0	$1/2\mu_0 c s_0^2 e$	$1.208\,664\,052\,189\times10^{14}$	Hz V^{-1}	not known
ratio $e/h = \frac{1}{2}K_J$	e/h	$1/\mu_0 c s_0^2 e$	$2.417\,328\,104\,378\times10^{14}$	A J^{-1}	-0.027
Josephson constant	K_J	$2/\mu_0 c s_0^2 e$	$4.834\,656\,208\,756\times10^{14}$	Hz V^{-1}	-0.027
elementary charge	e	$\sqrt{2\alpha A_0/\mu_0 c}$	$1.602\,176\,565\,000\times10^{-19}$	A s	0
Rydberg constant	R_∞	$m/8\mu_0 s_0^6 e^2$	$10\,964\,733.322\,649\,5$	m^{-1}	-0.082
Bohr radius	a_0	$\mu_0 s_0^4 e^2/\pi m$	$5.294\,666\,854\,026\times10^{-11}$	m	+0.055
Bohr magneton	μ_B	$\mu_0 c s_0^2 e^3/4\pi m$	$9.276\,546\,521\,260\times10^{-24}$	A m^2	+0.027
Nuclear magneton	μ_N	$\mu_0 c s_0^2 e^3/4\pi m_P$	$5.052\,165\,140\,176\times10^{-27}$	A m^2	+0.027

*Difference value in relation to the Committee on Data for Science and Technology, CODATA 2010

VII. The Josephson Constant and the Structural Constant of Atom

According to (20) and (58) we can write (q=e)

$$\nu = \frac{|e\Phi|}{2A_0}. \tag{90}$$

If we divide (90) with the potential $|\Phi|$, we obtain the ratio of two constants, which is again a constant. Using (88) we get conversion constant:

$$K_0 = \frac{\nu}{|\Phi|} = \frac{e}{2A_0} = \frac{e}{2\mu_0 c e^2 s_0^2} = \frac{1}{2\mu_0 c s_0^2 e}. \tag{91}$$

So, (90) we can write using (91):

$$\nu = K_0 |\Phi|. \tag{92}$$

Equation (92) is reminiscent of Josephson's equation of the inverse AC effect, $\nu = (2e/h)U_{DC}$, where $2e/h = K_J$ is Josephson's constant K_J, while U_{DC} is the voltage at the superconducting junction, analogous to potential $|\Phi|$ in (90). Using (88) we find

$$K_J = \frac{2e}{A_0} = \frac{2e}{\mu_0 c e^2 s_0^2} = \frac{2}{\mu_0 c s_0^2 e} = 4K_0. \tag{93}$$

VIII. The von Klitzing Constant and the Structural Constant of Atom

If we share the constant $A_0 = c\,\mu_0\, e^2 s_0^2$ expressed by (48), with e^2, we get again a constant:

$$R_K = \frac{A_0}{e^2} = \mu_0 c\, s_0^2. \tag{94}$$

This constant coincides with the von Klitzing's constant e^2/h which was obtained in the research of the quantum Hall effect, Table II. It should be noted that the achievement of either integer (1, 2, 3, …) or fractional values (1/3, 2/5, 3/7, 2/3, 3/5, 1/5, 2/9, 3/13, 5/2, 12/5, …) in this effect is reminiscent of our introduction the numbers $n^{\pm1}$.

IX. Conclusion

The paper describes the determination of the structural constant of atom s_0 and its application in the calculation of physical quantities. This constant is completely determined from Maxwell's equations in the framework of classical physics. Thus defined constant is clear, observable and measurable. This means that this constant is not necessary to further interpret.

The structural constant of atom helps in the interpretation of the fine-structure constant, and it can completely replace the fine-structure constant. The fine-structure constant has been

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proposed as a unit of isotope number named *boskovic* [B]. In that way one can classify all the currently known elements and their isotopes, which is now recorded more than 3 200.

In the paper Planck's constant, Josephson's constant, von Klitzing's constant and Rydberg constant theoretically are connected to the structural constant of atom s_0 .

The paper also shows limitations in the application of Planck's constant and proposes a solution for its increased use, through the replacement of Planck *h* with $A_0 = c\,\mu_0\,e^2 s_0^2$ and modification of the expression for the photon energy of $\Delta E_{em} = h\nu$ to $\Delta E_{em} = A_0\nu + mc^2 - \sqrt{(A_0\nu)^2 + (mc^2)^2}$.

This change leads to the correction of Duane-Hunt's law, through which could be carried out verification of the theory presented in this paper at voltages above 20 kV. The verification can also be done by using spectral analysis.

If the presented theory confirmed as correct, it would have a significant impact on the further development of science in general.

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