Multi-element Circuits Based on LCLC Resonant Tank
- Theory and Application

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Abstract: - The paper deals with novel of multi-element resonant circuits and its modification and application. Main circuits consist of series LC resonant branches and of parallel LC sinusoidal output filters. Review of multi-element circuits which contains more accumulation tanks is presented. Its mathematical description; design of accumulation elements and simulation experiments are given in main chapters of paper. Furthermore, mainly is focus given for application field and on possibility of its variable use. Multi-element circuits meet the requirements of the current market as are: small value of THD, high power density and very high efficiency. Also, such special circuits manifest inner self-regulation which provides resistance to short circuit. The paper also shows analysis of transient properties too. Base on the selected circuits are suggested control methods. All simulation results are verified by experimental measurement created on physical sample. On the end the paper is discussed the application of this circuits and its possible variable use in industry.

Key-Words: - multi-element circuits; LCLC; NF modulation; power electronic systems; transient analysis; non-linear; short-circuit proof system.

1 Introduction

Concept of the resonant converters had greatly expanded into various industrial and consumer applications, such as power supplies for distributed systems, electrical drives, laptops, LCD televisions as well as for aerospace, automotive and energy systems sector. Focusing on the development of power electronic semiconductors, the involvement of resonant converters into various applications is a must due to continual improvements in technology of power semiconductors manufacture. In order to improve electrical properties of power converters suited for power energy systems, the paper deals with investigation novel of multi-element resonant circuits and its modification and application. First, standard multi-resonant circuits are analyzed, as well as mathematical model is developed. After it, modifications of LCLC circuit for better dynamic behavior during start-up and short-circuit operation are described, whereby both operation modes have been verified and compared with standard solution.

The main criterion was achievement of low value of THD of output variables (voltage and current). Based on analysis, physical model was constructed, whereby key results are experimentally supported at the end of the paper. Each design is confirmed with precise measurements [1].

Investigation of a short circuit in multi-resonant network circuit is also included in the analysis. Proposed topologies are based on LCLC resonant circuits. The main focus is on its ability to withstand short circuit. During short circuit the output current is limited by the properties of resonant network which creates internal self-regulation. Is necessary, to determine appropriate control method in case of linear behaviour of the system. Each modification of standard LCLC converter is mathematically supported in order to understand basic design of multi-tank resonant converters for modern industrial applications [2], [3].

Fig.1 Trends course of development of power density [3]

A growing demand for saving energy and reducing the size of power systems have stimulated substantial research and development efforts towards high-efficiency and high-power density power supplies (Fig.1). The most effective way to achieve high power density in converters is to
increase the switching frequency so that the size of the passive components, such as the capacitor and inductor, as well as the transformer can be reduced, as they occupy a large portion of the overall size. Main design property of proposed converter is possibility to achieve low value of THD (below 5%) of output variables (voltage and current), at very high efficiency (over 97%). High system efficiency is one of the main quality indicators of power supplies. Therefore this parameter was investigated in wide region of switching frequency, in order to meet future demands on second quality factor – power density [3].

2 Multi-element Resonant Circuits

The group of resonant and quasi-resonant topologies consists of serial, parallel and serial-parallel resonant circuit. By combining the basic resonant circuits rise modified multi-elements resonant circuits. Resonant converters use two kinds of the switching technique: Zero voltage switching (ZVS) and Zero current switching (ZCS) [4]. Those techniques are now as soft switching. The converter can operate in ZVS and/or ZCS. The basic scheme of the resonant converter is given in the fig. 2 [12].

2.1 Comparison of selected multi-element topologies

One of the novel types of converters are LCLC converters based on LLC resonant scheme, and LCTLC inverter consisting of DC/DC buck converter LCLC resonant filter and HF transformer. The HF transformer can also be connected after the LCLC filter, if necessary, and can also be used to boost converter types. The inverter (LCTLC) is usually used as power supply for either HV rectifiers or HF cyclo-converters or matrix converters for 2-phase motor applications respectively [3], [5].

2.1.1 LCTLC circuit

The circuit is based on LCLC circuit where between serial (L₁, C₁) and parallel (L₂, C₂) accumulation tank is inputted HF transistor. It is feet by DC source and the shape of input voltage is switched in half-bridge connections [6]. In this case is output of the filter the HF harmonic waveform of the voltage (and current) is direct output mode with THD no more than 5% [5].

![Fig.3 Block scheme of the LCTLC circuit](image)

The scheme describes basic connection of the resonant converter composed by the DC source; switching network; resonant filter and multifunction output connected to load. Also, is possible to connect second stage converter (e.g. matrix converter). Is’s important to choose proper control method base on connected 2nd stage. Essence of the MRC’s concept is combining the positive properties of conventional topologies in one device. Multi-resonant circuit can absorb all of the parasitic elements. Therefore, enable operation with low switching losses at high switching frequencies [3], [3],[13].

![Fig.2 Block scheme of the resonant circuit](image)
Voltage gain characteristic for proposed LLC converter is shown in fig. 4. Principle, the shape is similar to standard LLC converter (left side of characteristic), whereby whole characteristic is clear combination of LLC and LCC converter (right side of characteristic). The circuit can be operated at higher or lower frequencies in ZVS region, achieving wide range of voltage gain [7].

2.1.2 LLCLC circuit
The circuit is well known and well described in the scientific literature. The resonant circuit is composed by serial-parallel LLC circuit and one parallel circuit. With one additional resonant element, a second band pass filter is created. A novel LLCLCL resonant tank is proposed as an example. The structure is similar to the previously proposed four element resonant tank, but an extra resonant inductor is inserted.

![Fig.5 Block scheme of the LLCLC circuit](image)

The half-bridge circuit is adopted as the primary-side structure. It is easy to extend to other types of input structures, including fullbridge, stacked half-bridge, and three-level structures. Similarly, it is easy to use other types of output structures, such as full-bridge, voltage-doubler and current-doubler structures.

The voltage gain of the proposed LLCLC resonant tank is illustrated in fig.6 (in range of load 10-100%). Conceptually, $L_r$, $C_r$ and $L_p$ contribute to the first band pass filter at low frequencies. The second band pass filter consists of $L_r$, $C_r$ and $C_p$, which dominate at high frequencies. The first band pass filter can help to deliver the fundamental component to the load. It functions as the traditional resonant converters [6]. The second band pass filter enhances the power delivery with utilization of higher harmonics. Consequently, with the injection of higher-order harmonics, the reactive power of the resonant tank can be reduced and lower RMS current and lower conduction loss can be achieved. The output signal contains higher harmonics what is increasing RMS value of rectified voltage on output. However, THD of output voltage (before rectifier) is higher due to injected higher harmonics [3].

2.1.3 LCL2C2 circuit
Based on previously analyzed multi-element topologies was created new resonant circuit LCL2C2. Circuit is composed by one serial resonant tank and two parallel resonant tanks. This circuit is proposed as non-isolated circuit with brought out zero leg. Block configuration of components in LCL2C2 is given in fig.5.

![Fig.7 Block scheme of the LCL2C2 circuit](image)

The "LCL2C2" circuit is one of the possible hybrid connections of resonant circuits. The main difference between the LCL2C2 and LCTLC converter is that the second one uses transformer to change the value of output voltage. In this kind of
circuit connection the control of output can be difficult. If the DC-AC inverter is considered, may be used the frequency or the asymmetric input control.

The multifunction output brings the possibilities for AC and DC output as well. Basically, output of the converter can be considered in three ways, i.e.:

a) direct AC output
b) diode rectifier
c) AC output with variable or constant frequency

Voltage transfer is identical to the LCLC circuit. Fig. 8 shows gain curves in dependency on the load change. Based on this characteristic it is able to determine the proper operation regions of LCL2C2 converter. Above resonant frequency, which is point of f\(_{\text{rel}}\) is equal 1, the region with ZVS conditions for switching transistor are achieved, whereby boarder between ZVS and ZCS region is limited by the peak gain values of each gain curve. Similar relation is valid below resonant frequency, whereby ZVS and ZCS region are mirrored compared to region above f\(_{\text{rel}}\) [12].

2.2 General Design of Accumulation Components

The resonant frequency of LC components should be the same as basic fundamental frequency of the converter and is governed by load requirements. Thus, based on the Thomson relation

\[
\omega_{\text{rez}} = \frac{1}{\sqrt{L \cdot C}}
\]

or, respectively

\[
L = \frac{1}{\omega_{\text{rez}} C}
\]

where \(\omega_{\text{rez}}\) is equal 2\(\pi\times\) fundamental frequency of the converter. Values of storage LC components and their parameters are important for properties of LCLC filter. Theoretically, \(\omega_{\text{rez}}L_1\) and other values of the converter can be chosen from a wide range [2], [6]. For our first design approximation we suppose a simple resonant circuit with a resonant frequency equal to the switching input frequency \((\omega_{\text{rez}} = \omega_{\text{sw}})\).

The LC design process can be considered from 3 different points of view or criteria:

1\(^{st}\): nominal voltage and current stresses at steady-states,
2\(^{nd}\): minimum voltage and current stresses during transients,
3\(^{rd}\): required value of total harmonic distortion of the output voltage.

In order to not exceed nominal voltages of the storage elements has been used value of internal impedance of the storage element equal to the nominal load \(|Z_N|\).

Let’s define the nominal design factor \(q_N\) for LC components as [4], [7]

\[
q_N = \frac{L\omega_{\text{rez}}}{|Z_N|} = \frac{1}{\omega_{\text{rez}} C |Z_N|}
\]

The above equation is similar to quality factor defined by \(q = \frac{L_{\text{load}} \omega_{\text{rez}}}{R_{\text{load}}}\), however \(q_N\) does not depend on the load \(R_{\text{load}}\).

The design formulas for LC accumulation elements can obtain:

\[
L = \frac{U_1^2}{\omega_1 P_1} q_N
\]

\[
C = \frac{P_1}{\omega_1 U_1^2} \frac{1}{q_N}
\]

The voltage on storage elements at nominal steady-state is defined as

\[
U_C = \frac{1}{\omega_{\text{rez}} C} l_N q_N = \frac{1}{\omega_{\text{rez}} C} U_1 q_N
\]

That means that for \(q_N\) equal to one, the voltages on storage elements will be nominal values, and are proportionally depend on \(q_N\) factor.

Going back to LCLC filter, then

\[
L_1 = \frac{U_1^2}{\omega_1 P_1} q_N
\]

\[
C_1 = \frac{P_1}{\omega_1 U_1^2} \frac{1}{q_N}
\]

\[
I_2 = \frac{U_1^2}{\omega_1 P_1} \frac{1}{C_2} = \frac{P_1}{\omega_1 U_1^2} q_N
\]

where \(U_1, P_1, \omega_1\) are nominal output voltage, power and frequency, respectively (fundamental harmonic)[3], [5].

2.2.1 Experimental Simulation of Multi-element Circuit

Simulation model was created according to the designed parameters of multi-element resonant circuits. MATLAB environment has been use to provide all the simulations experiments using suitable numerical method or directly preprogramed
functions. Time waveforms are given in following figures.

Fig. 9 Simulated waveforms of input and output voltage and current (per unit)

Based on theoretical assumptions, the system is operating in ZV/ZC mode. Waveforms of current and voltage on the switching transistor during operating process are given on fig. 4. The converter switches in zero voltage (ZVS) what is preferred operating area for the MOSFET transistors. ZVS conditions have been achieved moving the switching frequency above the resonant frequency.

Fig. 10 simulated waveforms of current and voltage on the switching transistor

Fig. 11 symmetrical output of LCL2C2 circuit

The waveforms in the fig. 11 are showing the output voltage and voltage on the both of branches with symmetric output of LCL2C2 circuit. The load voltage (also voltage on the branches) has harmonic shape whit low THD value. The simulated waveforms are matching with the theoretical assumptions. The determination of the THD value is given below.

Using Fast Fourier Transformation (FFS) was possible to calculate THD of output voltage. The resonant components of the filter are toned on basic harmonic; therefore the higher harmonic contents are suppressed.

Fig. 12 The harmonic content of LCLC converter

The total harmonic distortion (THD) was 5.59%. Because the simulation model considered with parasitic elements the value of THD raised over 5%.

3 Control methods of multi-resonant topologies

It is necessary, to determine appropriate control method in case of linear behavior of the system. Under the condition of non-linearity and taking parasitic into account, occurs the change of \( \frac{f_{res}}{f_{sw}} \) ratio.

It is possible to control output voltage by the classical frequency control method connecting corresponding kind of converter on the output side of system. Corresponding converter are rectifier or cyclo-convertor. This regulation is suitable for circuits with transformer. However, transformer brings additional losses to system. LCL2C2 circuit is transformer less circuit so; it may be considered other kind of control. By considering the different ways the inverter output can be choose the control method [8], [10].

3.1 Frequency ratio change control

One of the simples’ ways how to control resonant circuit is change of ration between switching and resonant frequency. Maximal gain of output voltage is when \( \frac{f_{res}}{f_{sw}} \) is equal1. Changing the switching frequency is possible to change magnitude of output signal. However, increasing the ration will grow number of harmonics contained in output signal.
3.2 Nonsymmetrical control method

The real output voltage of inverter waveform has a wide spectrum of harmonic components. Using nonsymmetrical control the output voltage of inverter (Fig. 13) comprises all harmonic components.

\[ U_{1M}(\beta) = \frac{4}{\pi} U \sin(\beta/2) \]

where \( U_{1M}(\beta) \) is magnitude of fundamental harmonic depend on pulse width and \( U \) is maximal value of output voltage. With growing \( \beta \) angle is raising number of even harmonics included in input signal.

![Fig.13 principal method of nonsymmetrical control](image)

Fig.13 principal method of nonsymmetrical control

Used resonant circuit is tuned on fundamental harmonic, but it should be tuned on switching frequency of converter as well. By the nonsymmetrical change of angle is possible to control magnitude of output voltage.

3.3 LF modulation of input voltage

One of possible way is to control output voltage on input side of converter. In this case would be voltage regulated prior to entering to resonant circuit. This is possible provide by bipolar PWM with LF modulation.

![Fig.15 NF modulation of output signal](image)

Fig.15 NF modulation of output signal

Basic harmonic HF signal with NF modulation passing resonant circuit (tuned to the HF signal-switching frequency) with unit gain and the output voltage is rectified (HF Schottky diode rectifier with respectively. MOSFET transistors in inverse mode) or modified (cycloconverter, matrix converter) and HF frequency signal component is removed by simple passive LC filter.

3.4 Multi-Resonant converter's inner self-feedback

Let’s consider LCLC circuit as common model for all multi-resonant circuits presented in the paper. Determine appropriate control method is necessary in case of linear behavior of the system. Under the non-linearity condition and taking parasitic into account, occurs the change of \( f_{res}/f_{sw} \) ratio. This change creates circuit’s inner self-feedback and it provides limitation of short circuit current. It causes saturation of magnetic elements and the change of the inductor inductances values. Therefore, the ratio between switching and resonant frequency is changing. Frequency ratio change impacts on the point of maximal gain. These phenomena can be considered the method of self-regulation due to own internal feedback. This phenomenon is caused by non-linearities of these circuits [11].

3.4.1 Multi-resonant non-linear circuitry

Modelling deals with Euler - and Taylor expansion methods for consequent numerical solution in Matlab environment. As an example, electrical circuit with serial rectifier diode [10], [11] includes the non-linearities as:

- input voltage
- non-linear capacitance of diode (important for PV applications)
• magnetic circuit of serial inductance and HF transformer magnetic circuit of transformer and parallel inductance

Created model of multi-resonant circuit was updated with nonlinear inductor. Mathematical method used in model nonlinear inductor was fictitious exciting functions method. More about this method is given in [11].

Fig. 16 Model of LCLC with nonlinear inductor (in state of saturation) [10]

The disadvantage of this regulation is that it causes the change in shape of output current and its THD increases by 3-5%. Also, is possible to consider over-dimensioned the accumulation elements where will saturation not occur.

Fig. 17 Voltage transfer of LCLC (parasitic and non-linear elements included in model)

Short circuit causes that output current increase and saturation of magnetic elements. Result is change of the inductor inductances values. Therefore, the ratio between switching and resonant frequency is changing. What is based on Thomson relationship (1). Also, change of frequency ration impacts on the point of maximal gain and moving resonant frequency (fig.17).

4 Transient Properties

Simulation model of multi-element circuit is built by applying knowledge of basic resonant circuits. Non-linear electronic elements as semiconductor devices and ferromagnetic inductors are included in model. Voltage transfer functions and impedance-frequency dependencies are theoretically derived, calculated, computationally simulated and analysed. Besides, the output voltage value does not depend on the load value.

Simulation model is based on the equations (1-7) for the design of accumulation components from previous chapter [8].

4.1 Analysis in frequency domain

Let's define nominal impedance for series resistive-inductive load

\[ |Z_N| = \sqrt{R_{\text{load}}^2 + (\omega L_{\text{load}})^2} = \frac{U_{\text{out}}}{P_{\text{out}} N} \] (9)

and nominal admittance for parallel resistive-inductive load

\[ |Y_N| = \sqrt{\left(\frac{1}{R_{\text{load}}}\right)^2 + \left(\frac{1}{\omega L_{\text{load}}}\right)^2} = \frac{P_{\text{out}} N}{U_{\text{out}}} \] (10)

On the beginning will be defined simple resistive load. Impedance of series and parallel part of the LCLC filter is defined by the following equations

\[ Z_1(\omega) = R_1 + j (\omega L_1 - \frac{1}{\omega C_1}) = \frac{R_1}{|Z_N|} |Z_N| j |Z_N| Q_1 (f_{\text{rel}} - \frac{1}{f_{\text{rel}}}) \] (11)

Where in \( R_1 \) is substitute the sum of resistance of series part of the filter (e.g. resistance of series filter coil; of filter capacitor; ...).

Thus

\[ \frac{|Z_1(\omega)|}{|Z_N(\omega)|} = \sqrt{r_1^2 + q_1 (f_{\text{rel}} - \frac{1}{f_{\text{rel}}})^2} \] (13)

Edited mathematical model of input impedance of LCLC looks:

\[ |Z_{\text{in}}(\omega)| = \left[ r_1 + \frac{(1+\frac{j}{DEN})^2}{DEN} \right]^2 + \left[ (f_{\text{rel}} - \frac{1}{f_{\text{rel}}}) q_1 - q_2 DEN \right]^2 \] (14)

where denominator marked DEN is defined as:

\[ DEN = \left(\frac{1}{r_2} + \frac{1}{r_1}\right)^2 + \left[ q_1 - q_2 DEN \right]^2 \] (15)

Impedance presentation in frequency domain in different states of load will be:

Fig.18 Filter input impedance vs. frequency in range 0-100% of load
Fig. 18 shows impedance dependence on frequency ratio. The ratio is composed by resonant frequency $f_{\text{res}}$ and switching frequency $f_{\text{sw}}$ what creates relative frequency. In point, where is $f_{\text{rel}}=1$ and load=0 impedance value grows to infinity. Impedance transfer is equal 1 where is $f_{\text{res}}/f_{\text{sw}}$ equal 1, what ensures that circuit operates in resonance. Also, there is possible to choose proper operation area.

As well as impedance transfer is possible to create voltage transfer model.

$$F(\omega) = \frac{|Z_2(\omega)|}{|Z_{\text{in}}(\omega)|} = \frac{1}{\sqrt{\text{DEN}}} \left\{ r_1 + \left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1/2} \right\}^2 \left[ \left( f_{\text{rel}} \frac{1}{r_1} q_{\text{rel}} - \frac{q_2}{\text{DEN}} \right) \right]^2 \quad (16)$$

Voltage transfer function of LCLC resonant circuit is given in fig. 7. The transfers curves are changing depend on the load (0-100%). However, in the resonance point ($f_{\text{res}}/f_{\text{sw}}=1$) is voltage transfer equal 1, what means that the system is no depend on load.

Fig.19 Voltage transfer function $U_2/U_1$ of LCLC filter in range 0-100% of load

Adding inductance into the final model, voltage transfer and impedance transfer for complex RL load will look:

Fig.21 Impedance transfer function $U_2/U_1$ of LCLC filter in range 0-100% of RL load;

4.1.1 Choosing proper operation area

Based on input previous analysis is possible to choose two izo-impedance (invariant impedance) operational points for switching frequency. In this case input impedance is not depending on the load of the inverter. Two mirror trajectories with minimal input impedance of the resonant circuit depending on the load. First point is when impedance is proportional depended on the load (fig.2). Similarly, voltage transfer frequency characteristic of the LCL2C2 resonant circuit offers two mirror trajectories with maximal output voltage of the circuit depending on the load, and also one point (A0) when the output voltage of the inverter does not depend on the inverter’s load. Also, is possible to determine the optimal operation frequencies for other value of overloading and functional relation is

$$|f_{\text{min}}|_{\text{overload}} = f(Z_{\text{overload}}) \quad \text{or} \quad |f_{\text{max}}|_{\text{overload}} = f(Z_{\text{overload}}), \quad (17)$$

to input current was be the same as nominal one. Carried-out results are original ones, and in spite of non-linear circuitry the output voltage THD is staying rather small, about 7-11 %.
In special operation states, multi-element circuits may present by inner self-feedback. This self-feedback provides limiting of short circuit current. It helps prevent destruction of the device [10].

5 Experimental verification
The paper deals with novel of multi-element resonant circuits. Higher discussed topologies and its properties were verified no physical samples. Experimental measurements fully respond theoretical assumptions and simulation experiments.

5.1 Voltage and current quantities of multi-resonant circuits
For MOSFET device is preferred operation area ZVS, what has been achieved during experimental measurement.

Based on the FFT analysis of output voltage we proceeded to calculate the real THD value. For this purpose we used next equation:

\[
THD \, (\%) = \frac{\sqrt{U_2^2 + U_3^2 + U_4^2 + U_5^2 + \cdots + U_n^2}}{U_{rms}} \times 100\%
\]

, where \( U_2, U_3, U_4, U_5, U_n \) are parts of higher harmonic order, and \( U_{rms} \) is root mean square value of output voltage. Based on this equation the computed THD value of output voltage of proposed converter is 4.02 [%].

5.2 Transient analysis experiments
Transient analysis was prepared on special physical sample –frequency tester.

The shape of transfer waveforms is similar to the simulation results (fig. 17). Output voltage values compared to simulation are similar too. The perceptual difference is from 3 till 15%. The biggest difference is in case of short circuit where the values in 80-90 kHz and between 110-120 kHz. Best match is observed at nominal load. Frequency ratio is changing as it was in simulation experiments where considered nonlinearity and parasitic elements was [9], [10].

These phenomena can be considered the method of self-regulation due to own internal feedback. In case of short circuit is output current limited by the converter self-regulation. Also, the regulation causes the current shape distortion and its THD increases about 3-5%. THD values about 6-9%. 
5.2.1 Multi-resonant non-linear experiments
Shape of voltage and current on the load during short circuit is shown on following picture.

![Figure 5: Output voltage and current quantities in state of short circuit (experimental)](image)

Fig. 5 Output voltage and current quantities in state of short circuit (experimental)

Experiments show that the real device is output current limited during short circuit. The shape has changed and THD value of output current and voltage increased. This impacts to the transfer properties and creates inner self-feedback.

Experimental measurements show that all mathematical models and theoretical assumptions analyzed in article was successfully verified.

6 Conclusion
In the paper was discussed about multi-resonant circuits’ their theory and application. Selected circuits are described and analyzed in second chapter. All tree topologies are based on LCLC circuit. LCTLC and LLCLC are general know circuits. However, LCL2C2 can be considered as new multi-resonant circuit with many advantages. Specific control methods are briefly analyzed as possible way to regulate output voltage of these circuits. In particular, LF modulation of input voltage appears to be promising way to control output voltage, especially for LCL2C2 circuit. Base on mathematical models of those multi-resonant circuits was carried out transient analysis. Circuits have been investigated in different stages of load (in wild range). Under the obtained results was possible to choose proper operation areas of multi-resonant circuits. Everything is depends on design and final application. Simulation results shown, that LCL2C2 circuit has inner self-feedback. This feedback provides prevention against short circuit. To understand better this method of self-protection was necessary to create system with non-linear elements. Using fictitious exciting functions method was possible to simulate this system. Under the non-linear condition occurs the change of $f_{res}/f_{sw}$ ratio. It provides limitation of short circuit current. It causes saturation of magnetic elements and the change of the inductor inductances values.

Theory, models and simulation result was verified by experimental measurements provide on physical samples prepared in our laboratories. All simulation, including non-linearity and inner self-feedback were confirmed by real experiments. The article can serve as a guide for the analysis of multi-resonant circuit.

References:

