Systems Optimization Prospected from Torus Cyclic Groups

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Abstract—This paper relates to techniques for improving the quality indices of engineering devices or systems with respect to performance reliability, transmission speed, positioning precision, and resolving ability, using novel design based on structural perfection and remarkable properties of proposed modification of combinatorial configurations, prospected from fundamental laws of the space dimensionality, namely the concept of Ideal Vector Rings (IVR)s. These design techniques make it possible to configure systems with fewer elements than at present, while maintaining or improving on resolving ability and the other significant operating characteristics of the system.

Keywords—Combinatorial configuration, systems optimization, perfect torus group, monolithic code, vectorial space harmony laws.

I. Introduction

Combinatorial configurations arise in many problems of pure mathamatics, notably in algebra, applied physics, topology, and geometry. Combinatorics also has many applications in mathematical optimization, computer science, and quantum physics. One of the most acceptable parts of combinatorics is systems theory, which also has numerous natural connections to other areas. Combinatorial optimization started as a part of combinatorics and graph theory, but is now viewed as a branch of applied mathematics and computer science, related to coding theory as a part of design theory with combinatorial constructions of error-correcting codes. The main idea of the subject is to design efficient and reliable methods of data transmission. It is a large field of study, part of information theory in systems engineering and data communications. Combinatorial configurations such as cyclic difference sets [1] and Ring Bundles [2], is known, to be of very important in systems engineering for improving the quality indices of devices or systems with non-uniform structure (e.g. arrays of radar systems) with respect to resolving ability [3].

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II. Optimum Ordered Combinatorial Sequences

A. Optimum Chain Ordered Sequences

The “ordered chain” approach to the study of elements and events is known to be of widespread applicability, and has been extremely effective when applied to the problem of finding the optimum ordered arrangement of structural elements in a distributed technological systems.

Let us calculate all S_n sums of the terms in the numerical n-stage chain sequence of distinct positive integers $C_n = \{k_1, k_2, \ldots, k_n\}$, where we require all terms in each sum to be consecutive elements of the sequence. Clearly the maximum such sum is the sum $S_c = k_1 + k_2 + \ldots + k_n$ of all n elements; and the maximum number of *distinct* sums is

$$S_n = 1 + 2 + \ldots + n = n\,(n+1)/2 \qquad (1)$$

If we regard the chain sequence C_n as being *cyclic*, so that k_n is followed by k_1, we call this a *ring sequence*. A sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length (number of terms) from 1 to n -1. In addition, there is the sum S_n of all n terms, which is the same independent of the starting point. Hence the maximum number of distinct sums S_{max} of consecutive terms of the ring sequence is given by

$$S_{max} = n\,(n-1) + 1 \qquad (2)$$

Comparing the equations (1) and (2), we see that the number of sums S_{max} for consecutive terms in the ring topology is nearly double the number of sums S_n in the daisy-chain topology, for the same sequence C_n of n terms.

B. Two-dimensional Vector Rings

Let us calculate all S sums of the terms in the n-stage ring sequence of non-negative integer 2-stage sub-sequences (2D vectors) of the sequence $C_{n2} = \{(k_{11}, k_{12}), (k_{21}, k_{22}), \ldots, (k_{i1}, k_{i2}), \ldots, (k_{n1}, k_{n2})\}$ as being *cyclic*, so that (k_{n1}, k_{n2}) is followed by (k_{11}, k_{12}), where we require all terms in each *modular* 2D *vector sum* to be consecutive elements of the *cyclic sequence*, and a modulo sum m_1 of k_{i2} and a modulo sum m_2 of k_{i2} are taken, respectively. A *modular 2D vector sum* of consecutive terms in this sequence can have any of the n terms as its starting point, and can be of any length (number of terms) from 1 to n-1. Hence the maximum number of such sums is given by

$$S = n\,(n-1) \tag{3}$$

If we require all *modular vector sum* of consecutive terms give us each vector value exactly R times, than

$$S_R = \frac{n(n-1)}{R} \tag{4}$$

Let $n = m_1$, $n - 1 = m_2$, then a space coordinate grid $m_1 \times m_2$ forms a frame of two modular (close-loop) axes modulo m_1 and modulo m_2, respectively, over a surface of torus as an orthogonal two modulo cyclic axes of the system being the product of two (t=2) circles. We call this two-dimensional Ideal Vector Ring (2D IVR), shortly "Vector Ring".

Example: Let n=3, m_1=2, m_2 =3, R=1, and complete set of the IVRs takes four variants as follows:

(a) {(0,1),(0,2),(1,2)}; (b) {(0,1),(0,2),(1,1)};
(c) {(0,1),(0,2),(1,0)}; (d) {(1,0),(1,1),(1,2)}.

To see this, we observe that *ring* sequence {(0,1), (0,2), (1,2)} gives the next circular vector sums to be consecutive terms in this sequence:

$$\left.\begin{aligned}(0,1)+(0,2)&=(0,0)\\(0,2)+(1,0)&=(1,2)\\(1,2)+(0,1)&=(1,0)\end{aligned}\right\}(\operatorname{mod}2,\operatorname{mod}3) \tag{5}$$

So long as the terms (0,1), (0,2), (1,2) of the three-stage (n=3) ring sequence themselves are two-dimensional vector sums also, the set of the modular vector sums (m_1=2,m_2=3) forms a set of nodal points of annular reference grid over 2×3 exactly once (R=1):

(0,0)	(0,1)	(0,2)
(1,0)	(1,1)	(1,2)

Easy check to see, that the rest of these ring-sequences has the principal property of forming reference grid 2×3 over a torus using only three (n=3) two-stage (t=2) terms of these ring sequences.

Schematic model of two-dimensional Vector Ring in torus system of reference is given below (Fig.1) as the simplest and well useful for analytic study of two-dimensional Vector Rings.

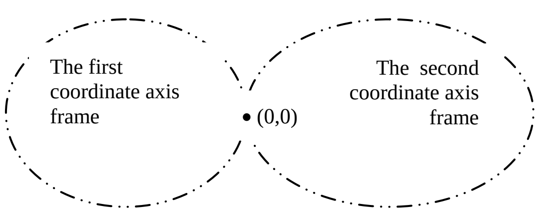

Fig.1. Schematic model of two-dimensional Vector Ring in torus system of coordinates with ground coordinate (0,0).

Easy check to see, that the rest of ring sequences have the principal property of forming reference grid 2×3 over a torus using only three (n=3) two-stage (t=2) terms of these circular sequences.

III. Multidimensional Vector Cyclic Groups

A. Principal Consideration

To discuss concept of Vector Cyclic Groups (VCG)s let us regard structural model of t-dimensional vector ring as ring n-sequence $C_{nt} = \{K_1, K_2, .., K_i, ..., K_n\}$ of t-stage sub-sequences (terms) $K_i = (k_{i1}, k_{i2}, ..., k_{it})$ each of them to be completed with nonnegative integers (Fig.2).

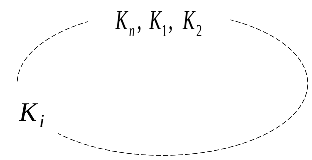

Fig.2. Schematic model of t-dimensional n-stage ring sequence.

Here is an example of 3D Vector Ring with n= 6, m_1=2, m_2=3, m_3=5, and R=1 which contains circular 6-stage sequence of 3-stage (t=3) sub-sequences $\{K_1, ..., K_6\}$: $K_1 \Rightarrow (k_{11}, k_{21}, k_{31})$= (0,2,3); $K_2 \Rightarrow (k_{12}, k_{22}, k_{32})$ = (1,1,2); $K_3 \Rightarrow (k_{13}, k_{23}, k_{33})$ = (0,2,2); $K_4 \Rightarrow (k_{14}, k_{24}, k_{34})$ = (1,0,3); $K_5 \Rightarrow (k_{15}, k_{25}, k_{35})$ = (1,1,1); $K_6 \Rightarrow (k_{16}, k_{26}, k_{36})$ = (0,1,0). The set of all circular sums over the 6-stage sequence, taking 3-tuple (t=3) modulo (2,3,5) gives the next result:

(0,0,1)=((0,2,2) + (1,0,3) + (1,1,1)),
(0,0,2)=((1,1,2) + (0,2,2) + (1,0,3)),
(0,0,3)=((0,2,3) + (0,1,0)),
(0,0,4)=((0,2,2) + (1,0,3) + (1,1,1) + (0,1,0) + (0,2,3)),
(0,1,1)=((0,2,2)+ (1,0,3) + (1,1,1) + (0,1,0)),
(0,1,2)=((1,0,3)+ (1,1,1) + (0,1,0) + (0,2,3)),
(0,1,3)=((1,1,1)+ (0,1,0)+ (0,2,3)+(1,1,2)+ (0,2,2)),
(0,1,4)=((0,1,3) + (1,1,1)),
(0,2,0)=((0,2,3)+ (1,1,2) + (0,2,2) + (1,0,3)),
(0,2,1)=((1,1,1)+ (0,1,0) + (0,2,3) + (1,1,2)),
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(1,2,4)=((0,2,3)+(1,1,2)+(1,1,1)+(1,0,3)+(0,1,0)).

Easy to see this verify of the theoretical proposition (6).

$$\prod_{1}^{t} m_i = \frac{n(n-1)}{R}\,, \qquad (m_1, m_2, \ldots, m_t)=1 \tag{6}$$

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B. Vector Ring Sequences as Cyclic Groups

Next, we consider a set of Vector Rings with n=3, m_1=2, m_2 =3, R=1 as a cyclic multiplicative group of a finite field. With this aim let us multiply the VR {(0,1),(0,2),(1,2)} trough by 2D (t=2) coefficient (1,2) taking both (mod 2), and (mod 3) as follows: (0,1)·(1,2) ⇒ (0,2), (0,2)·(1,2) ⇒ (0,1), (1,2)·(1,2) ⇒ (1,1). As a result of this transformation we got circular sequence {(0,2),(0,1),(1,1)} different from the previous but it is the same as the sequence (b), and the reverse transform by the multiplicative coefficient is true. However, multiplying circular sequences (b) or (c) through by (1,2) no transform them to others variants of the sequences but to themselves as combinations of reflection and cyclic shifting. Hence, the complete set of four VRs with n=3, m_1=2, m_2 =3, and R=1 contains both two isomorphic, and two non-isomorphic variants of the sequences, each of them makes it possible to cover the set of nodal points over torus grid 2×3 exactly once (R=1) using only three (n=3) basic vectors for configure optimum specify coordinates with respect to torus surface frame of reference. A new type of cyclic groups is among the most perfect Vector Rings which properties hold for the same set of the VRs in varieties permutations of terms in the set (e.g. set of two-dimensional Vector Rings {(1,0), (1,1), (1,2), (1,3), (1,4)} and {(1,0), (1,2), (1,4), (1,1), (1,3)}. We call this class of Vector Rings the "Perfect Torus Group" or "Gloria to Ukraine Stars". We have found numerous families of the Stars.

Here is the simplest and well useful for analytic study and applications of the underlying properties of Torus and Hypertorus Groups for development of new mathematical, physical and technological results.

IV. Optimum Monolithic Vector Codes

A. Useful Properties of Optimum Vector Codes

The remarkable properties of Vector Ring Sequences are that all ring sums of vectors in the sequence exhaust the set of vectors of a finite modular vector space by R ways exactly, which allows on binary encoding of two- and multidimensional vectors as sequences of the same signals or characters in ring code combination length. This makes it possible to use *a priori* maximal number of combinatorial varieties of ring sums for coded design of signals (6). As an example it is chosen the VR sequence {(1,1), (0,1), (2,2), (2,1)} with n=4, m_1=3, m_2=4, R=1. Here digit weight of the first position is vector value (1,1), the next – (0,1), (2,2), and (2,1) formed a circle. Here is result of the code design (Table 1).

TABLE 1.
Vector code based on the VR {(1,1), (0,1), (2,2), 2,1)}

Vector	Code	Vector	Code	Vector	Code
(0,0)	1110	(1,0)	0111	(2,0)	1011
(0,1)	0100	(1,1)	1000	(2,1)	0001
(0,2)	1000	(1,2)	1100	(2,2)	0010
(0,3)	1101	(1,3)	0011	(2,3)	0110

We can see that sequence {(1,1),(1,0),(0,2),(1,1)} forms complete set of ring code combinations on 2D ignorable array 3×4, and each of its occurs exactly once (R=1). Note, each of them forms massive arranged (solid parts of bits) both symbols "1" and of course "0" in the all possible binary circular code combinations. This property makes VR codes useful in applications to coded design of signals for communications, control systems and vector computing with a limited number of bits and improving noise immunity.

B. Definitions of the Ring Monolithic Vector Codes

a) Ring Monolithic Code is a set of ring sequence code combinations which the same characters arranged all together into the code combinations.

b) Numerical Optimum Ring Code is weighted binary Ring Monolithic Code which ring n- sequence of positive integer digit weights forms a set of binary n-digital code combinations of a finite interval [1,S], the sums of connected digit weights taken modulo S=n(n-1)/R enumerate the set of integers [1,S] exactly R-times.

c) Two-dimensional Optimum Ring Code is weighed binary Ring Monolithic Code which set of connected 2-stage modular sums taken modulo m_1 and m_2, respectively, allows an enumeration of nodal points of reference grid $m_1 \times m_2$ over torus exactly R-times with respect to torus surface frame of axes, $m_1 \cdot m_2 = n(n-1)/R$.

d) Multidimensional Optimum Ring Code is weighed binary Ring Monolithic Code which set of connected t-stage modular sums taken modulo m_1, m_2 ,…, m_t , respectively, allows an enumeration of nodal points of reference grid $m_1 \times \ldots \times m_t$ over hypertorus exactly R-times, $m_1 \cdot m_2 \ldots \cdot m_t = n(n-1)/R$.

V. Conclusion

Concept of the systems optimizations provides, essentially, a new model of technical systems. Moreover, the optimization has been embedded in the underlying combinatorial models. The favorable qualities of the Ideal Vector Rings provide breakthrough opportunities to apply them to numerous branches of science and advanced technology, with direct applications to vector data coding and information technology, signal processing and telecommunications, and other engineering areas. Structural perfection and harmony has been embedded not only in the abstract models but in vectorial laws of the real world [4], [5].

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Current research interests: The scientific basis of multidimensional optimum distributed systems theory, including the appropriate algebraic structures based on cyclic groups in extensions of Galois fields, and the generalization of these methods and results to optimization of a larger class of technological systems; development of fundamental and applied research in systems engineering for improving such quality indices as reliability, precision, speed, resolving ability, and functionality, using innovative methodologies based on combinatorial techniques; better understanding of the fundamental role of summery and asymmetry relationships in the worldwide harmony laws. Previous research interests: Design and engineering of an improved devices and process engineering for industrial automation of power stations.

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