PMSG wind system control for time-variable wind speed by imposing the DC Link current

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Abstract—This paper presents a method for controlling a wind system, - wind turbine (WT) + permanent magnet synchronous generator (PMSG) - so as to reach an optimal energetic operation at a time-variable wind speed. Wind speed and momentary mechanical angular speed of the PMSG impose the generator load value in the energetically optimal region. By energy balance measurements made with speed and power measurements, the generator load is determined so that the system has been brought into the energetic optimal region. It analyzes the maximum power operation in a WT by changing the load to the generator, while the wind speed significantly varies over time. The coordinates of the maximum power point (MPP) changes over time and they are determined by the values of the wind speed and mechanical inertia. Not always wind system can be lead in a timely manner in the MPP. The speed variation of the wind speed and the inertia value are two fundamental elements on which the MPP operation depends. By prescribing the amount of DC link current, Icc, the main circuit of the converter can achieve a simple and useful system tuning WT + PMSG. Operation control method in the optimal energy region of the WT is based on the knowledge of the Icc current value, which is determined by wind speed and momentary mechanical angular speed, MAS.

Keywords—Permanent Magnet Synchronous Generator, mathematical model of Wind Turbine, maximum power point, wind system.

I. INTRODUCTION

In the literature [1-21] various mathematical models of wind turbines (MM-WT) offered by building companies and/or obtained under laboratory conditions are presented, far different from those in real conditions operation [7, 12, 19]. For this reason the final result, especially the obtained electrical energy has a value less than the maximum possible at the maximum power point (MPP) operating at optimal mechanical angular speed (MAS). In most works is treated the operation of the wind turbine (WT) at MPP. [3, 5, 11, 21]. In some cases, [7, 9, 15, 11, 21], there are used mathematical models which are only partially valid, because of the continuous varying weather conditions. The laboratory conditions where they have obtained the turbine characteristics are different from those in real operation [11, 15, 17].

Recent works [1, 2, 3, 4] use control algorithms based on the measurement of wind speed and prescribing optimal speed of the mechanical angular speed in the MPP region. The estimation of the optimal MAS on the basis of the wind speed is a complex problem solved by mathematical calculations and with specialized simulation software [2, 3, 5].

Method of bringing the wind system operating point in the MPP region, by appropriately modifying the electric generator load requires the measurement of the wind speed and is quite powerful, [17,19,21], in certain circumstances. It can analyze these variations in time by knowing the wind speed and given the values of the moments of inertia.

There are geographical areas where the wind speed changes its value in less time [8, 9, 17]. In Romania, the wind speed varies in time and therefore the method can be applied in certain areas only after a prior study.

The method is based on the dependency of the power of WT on MAS, that means the function $P_{WT}(w)$ has, at a certain speed, a maximum value for MAS, $\omega_{OPTIM}$ (Fig. 1).

For wind speed which does not change his value over time, the operation in the MPP region can be performed quite simply. For wind speeds which significantly vary over time, the problem becomes complex and sometimes unsolvable (if the wind quickly changes the speed).

Analysis of the MPP operation is done by simulation using specific mathematical models for WT and PMSG. By changing the PMSG load, the system try to reach the MPP region and the transient phenomena can be visualized by solving the movement equation WT+PMSG system.
II. THE MATHEMATICAL MODEL OF THE WIND TURBINE

We will use a classical turbine model [14], which allows the estimation of the reference angular speed \( w_{\text{ref}} \). The mathematical model of the WT allows also the calculation of the optimal speed, so as the captured energy will be a maximum one.

The power given by the WT can be calculated using the following equation:

\[
P_{WT} = \rho \pi R_p^2 C_p(\lambda) V^3
\]

where: \( \rho \) - is the air density, \( R_p \) – the pales radius, \( C_p(l) \) – power conversion coefficient, \( l = R_w/V \), \( V \) - the wind speed, \( w \) – mechanical angular speed (MAS).

The power conversion coefficient, \( C_p(l) \), could be calculated as follows:

\[
C_p(\lambda) = c_1 \left( \frac{c_2 - c_3}{\lambda} \right) e^{\frac{c_4}{\lambda}}, \quad \frac{1}{\lambda} = \frac{1}{\lambda_0} - 0.0035,
\]

\[
c_1 - c_4 \text{ are data-book constants.}
\]

By replacing, we can obtain the the power conversion coefficient as follows:

\[
P_{WT}(\omega, V) = \rho \pi R_p^2 C_p(\lambda) V^3 = 1.225 \pi 1.5^2 c_1 \left( \frac{V}{1.5 \lambda_0} - 0.0035 \right) - c_3 e^{-c_4 \left( V/1.5 \lambda_0 - 0.0035 \right)} V^3
\]

And the power given by the wind turbine can be calculated as follows:

\[
P_{\text{WT}}(\omega, V) = \rho \pi R_p^2 C_p(\lambda) V^3 = 1.225 \pi 1.5^2 c_1 \left( \frac{V}{1.5 \lambda_0} - 0.0035 \right) - c_3 e^{-c_4 \left( V/1.5 \lambda_0 - 0.0035 \right)} V^3
\]

or

\[
P_{\text{WT}}(\omega, V) = \rho \pi R_p^2 C_p(\lambda) V^3 = 1.225 \pi 1.5^2 c_1 \left( \frac{V}{1.5 \lambda_0} - 0.0035 \right) - c_3 e^{-c_4 \left( V/1.5 \lambda_0 - 0.0035 \right)} V^3
\]

By replacing this result, it yields:

\[
P_{\text{WT-MAX}}(V) = k_p \cdot V^3
\]

This result proves a cubic dependency of the WT power on the wind speed.

If the wind speed has large variations, this result must be reanalyzed.

The mathematical model of the PMSG

To analyze the behavior of the system WT-PMSG for the the time-varying wind speeds, it uses orthogonal mathematical model for permanent magnet synchronous generator (PMSG) given by the following equations [5]:

\[
\begin{align*}
-U \sqrt{3} \sin \theta &= R_1 I_d - \omega L_d I_q \\
U \sqrt{3} \cos \theta &= R_1 I_q + \omega L_d I_d + \omega \Psi_{PM} \\
T_{\text{PMSG}} &= p_1 \left( I_d - L_q \right) I_d q + I_q \Psi_{PM}
\end{align*}
\]

where: \( U \) – stator voltage, \( L_q \), \( L_d \) – d-axis and q-axis stator currents, \( \theta \) – load angle, \( R_1 \) – phase resistance of the generator; \( L_d \) - synchronous reactance after d axis; \( L_q \) - synchronous reactance after q axis; \( \Psi_{PM} \) - flux permanent magnet; \( T_{\text{PMSG}} \) - PMSG electromagnetic torque.

III. OPERATING CONTROL IN THE MPP REGION

The study of operation in the MPP region will be performed by simulation using the following mathematical models.

The mathematical model for the WT (MM-WT)

For the wind turbine, the producer provides the experimental power characteristics [14], \( P_{\text{WT}}(w, V) \)

\[
P_{\text{WT}}(\omega, V) = 1191.5 \cdot (V/\omega - 0.02) \cdot e^{-98.06 \cdot (V/\omega)} \cdot V^3
\]

The mathematical model for the PMSG (MM-PMSG)

From the nominal values of the PMSG [1], for the nominal power: \( P_N = 5 \text{[kW]} \), it yields \( R_1 = 1.6 \text{[W]} \), \( L_d = 0.07 \text{[H]} \), \( L_q = 0.08 \text{[H]} \), \( \Psi_{PM} = 1.3 \text{[Wb]} \).

From the equations of the PMSG, it obtains

\[
\begin{align*}
\frac{d}{d \omega} \left( 1191.5 \cdot (V/\omega - 0.02) \cdot e^{-98.06 \cdot (V/\omega)} \cdot V^3 \right) &= 0 \\
\omega_{\text{ref}} &= 31.115 \cdot V
\end{align*}
\]

For this value of MAS, the maximum power is obtained:

\[
P_{\text{WT-MAX}} = 0.61884 \cdot V^3
\]

The mathematical model for the PMSG (MM-PMSG)

From the nominal values of the PMSG [1], for the nominal power: \( P_N = 5 \text{[kW]} \), it yields \( R_1 = 1.6 \text{[W]} \), \( L_d = 0.07 \text{[H]} \), \( L_q = 0.08 \text{[H]} \), \( \Psi_{PM} = 1.3 \text{[Wb]} \).

From the equations of the PMSG, it obtains

\[
\begin{align*}
-R_1 I_d &= 1.6L_d \omega - 0.08 L_q \\
-R_1 I_q &= 1.6L_q + 0.07 L_d + \omega \Psi_{PM} \\
T_{\text{PMSG}} &= -0.01 \cdot I_d L_q + I_q \Psi_{PM} \\
\Psi_{PM} &= 1.3 \\
P &= \left( I_d^2 + I_q^2 \right)
\end{align*}
\]
### 3.1. Case study for time-variable wind speed

For a sinusoidal time-variable wind speed, as presented in Fig. 2, with \( T = 35\,[\text{s}] \):

\[
V(t) = (16 - 6 \cdot (\sin 0.17943t))e^{-t/3600}
\]  

\( \text{Fig. 2. Time variation of the wind speed} \)

The wind speed is continuously monitored and the equivalent wind speed and the optimum DC link current are calculated at discrete time intervals \( \Delta t = T \).

The value of the DC link current \( I_{cc} \) is also continuously monitored, depending on the error:

\[
\Delta I = I_{cc} - I_{cc,\text{OPTIM}}
\]

the load resistance \( R \) is consequently modified.

The control of the wind system is realized based on the two measurements, presented above:

1. Wind speed
2. Current \( I_{cc} \)

Using [1], for the time interval \( \Delta t = [a, a+T] \) we can define an equivalent wind speed, as follows:

\[
V_{ECH} = \frac{3.56}{\sqrt{T}} \int_{a}^{a+T} \left( 16 - 6 \cdot (\sin 0.17943t) \right) e^{-t/3600} \, dt
\]  

With a period of \( 35\,[\text{s}] \), optimal MAS is calculated starting from \( t=40\,[\text{s}] \) (i.e. 40, 75, 110 … [s]), using the dependency:

\[
\omega_{\text{OPTIM}} = 31.817 \cdot V_{ECH}
\]

The following results are obtained:

- For the interval \( \Delta t = 5+40 \,[\text{s}] \), \( V_{ECH} = 17.187 \,[\text{m/s}] \) and \( \omega_{\text{OPTIM}} = 546.84 \,[\text{rad/s}] \)
- For the interval \( \Delta t = 40+75 \,[\text{s}] \), \( V_{ECH} = 17.021 \,[\text{m/s}] \) and \( \omega_{\text{OPTIM}} = 541.56 \,[\text{rad/s}] \)
- For the interval \( \Delta t = 75+110 \,[\text{s}] \), \( V_{ECH} = 16.856 \,[\text{m/s}] \) and \( \omega_{\text{OPTIM}} = 536.31 \,[\text{rad/s}] \)

#### 3.1.1. The control system by imposing the current \( I_{cc} \)

The power acquired by the PMSG is found in the intermediate circuit power and, from this equation, the \( I_{cc} \) current is obtained. \([1]\) and \((21)-(25)\)

\[
I_{cc} = \frac{P_{\text{PMSG}}}{U_{cc}} = \left( \frac{4225R\omega_{0}^2 - 4\omega_{0}^2 + 625R_{0}^2 + 2000R + 1600}{(125R_{0}^2 + 4000R + 3200 + 7\omega_{0}^2)^2} \right)/500
\]

#### Time evolution of the process

The simulations are based on the mechanical equation:

\[
J \frac{d\omega}{dt} = T_{\text{WT}} - T_{\text{PMSG}}
\]

where \( J \) is equivalent inertia moment, \( T_{\text{PMSG}} \) is the torque of the PMSG, \( T_{\text{WT}} \) is the torque of WT. By imposing the conduction angle of the converter between PMSG and the network, different values for load resistance and thus for the current \( I_{cc} \) are obtained.

The system is lead in the optimal energy region by imposing a DC link current, as results from energy balance, presented below:

To obtain the optimum MAS, \( \omega_{\text{OPTIM}} \), the PMSG load must be adjusted based on:

- kinetic energy variations of the moving parts
- optimum MAS to be reached at the moment \( t=45\,[\text{s}] \)

From the mechanical equation, it yields:

\[
J \frac{d\omega}{dt} = \omega \cdot T_{\text{WT}} - \omega \cdot T_{\text{PMSG}}
\]

\[
J \cdot (\omega_r^2 - \omega_{k-1}^2)/2 = \int_{k-1}^{k} P_{\text{WT}} \cdot dt - \int_{k-1}^{k} P_{\text{PMSG}} \cdot dt
\]

The energy to be captured by the PMSG, during \( \Delta t = t_k - t_{k-1} \) time interval is:

\[
W_{\text{PMSG}} = \int_{k-1}^{k} P_{\text{PMSG}} \cdot dt = \int_{k-1}^{k} P_{\text{WT}} \cdot dt - J \cdot (\omega_r^2 - \omega_{k-1}^2)/2 = E(\Delta t) - J \cdot (\omega_r^2 - \omega_{k-1}^2)/2
\]

Where \( E(\Delta t) \) is the value of energy to be captured during \( \Delta t \) time interval. It has two components:

1. \( \int_{k-1}^{k} P_{\text{WT}} \cdot dt \) – energy captured by the wind turbine
2. \( J \cdot (\omega_r^2 - \omega_{k-1}^2)/2 \) – rotational kinetic energy

The control process has two steps:

**Step 1:** bringing the system in the optimum energetic region

**Step 2:** keeping the system in the optimum energetic region

Could be done in two ways:

a. By loading the generator at maximum power if the initial MAS is greater than the optimum value

b. By no-load operation if the initial MAS is less than the optimum value

#### a. PMSG loading at maximum admissible power:

Starting from an initial speed \( \omega(0) = 555 \,[\text{m/s}] \), from \((a1)\) we can obtain min and max values for WT power.
It is necessary to bring the WT at optimal speed and only after connect the generator to the grid.

b. **Generator operates at no-load**
   - Measurement of wind speed and calculation of \( \omega_{\text{OPTIM}} \);
   - Measurement of MAS and comparison with \( \omega_{\text{OPTIM}} \);
   - When \( \omega = \omega_{\text{OPTIM}} \), the generator is connected to the grid.

After calculations, the following values are obtained:

\[
\begin{align*}
\omega_{\text{OPTIM}-40} & = 546.84 \, \text{[rad/s]} \\
P_{\text{PMSG}-40} & = 7075.9 \, \text{[W]} \\
R & = 453.85 \, \text{[W]}
\end{align*}
\]

**Step 2**: keeping the system in the optimum energetic region

Load at \( t=75 \, \text{[s]} \)

The energy captured by the PMSG, \( W_G \), in the interval \( Dt = 40+75 \, \text{[s]} \) can be estimated by measuring the electrical energy during this interval or, by simulations, from the mechanical equation and using the PMSG power.

During this interval, the variation of the kinetic energy is:

\[
W_{\text{KINETIK-REAL}} = J \cdot (\omega^2(75) - \omega^2(40)) / 2 = -6778.9 \, \text{[J]} \tag{32}
\]

The electric energy captured by the PMSG, during the same interval, is:

\[
W_G(35) = 2.4710 \times 10^5 \, \text{[J]} \tag{33}
\]

The wind energy captured by the wind turbine is:

\[
E(35) = 2.4028 \times 10^5 \, \text{[J]} \tag{34}
\]

It can prove the conservation of energy, with a very small error (\( \approx 10^{-2} \% \)).

**Remark 1**: Practically, based on the variations of kinetic energy and energy captured by the PMSG, the wind energy can be obtained.

To reach optimum MAS

\[
\omega_{\text{OPTIM}-75} = 541.56 \, \text{[rad/s]} \tag{35}
\]

it would be necessary a load for the generator calculated from energy equation:

Required kinetic energy:

\[
W_{\text{KINETIK-REQ}} = J \cdot (\omega^2_{\text{OPTIM-75}} - \omega^2(40)) / 2 = -1.1494 \times 10^5 \, \text{[J]} \tag{36}
\]

Wind energy captured in this time interval:

\[
E(35) = W_G(35) + W_{\text{KINETIK-REAL}} = 2.4032 \times 10^5 \, \text{[J]} \tag{37}
\]

The required energy for the PMSG is:

\[
W_{\text{PMSG-REQ}}(35) = E(35) - W_{\text{KINETIK-REQ}} = 3.5522 \times 10^5 \, \text{[J]} \tag{38}
\]

By estimation a medium power during this interval,

\[
P_{\text{PMSG-MED}} = W_{\text{PMSG-REQ}}(35) / 35 = 10149 \, \text{[W]} \tag{39}
\]

Using power equation and with \( w = 544.2 \), the required load to reach the optimal region is:

\[
R_{\text{PMSG-REQ-75}} = 311.64 \, \text{[\Omega]} \tag{40}
\]

In these conditions, the power to be prescribed to the PMSG (\( P_{\text{PMSG-P-75}} \)) is:

\[
P_{\text{PMSG-P-75}} = 10001 \, \text{[W]} \tag{41}
\]

**Remark 2**: The captured wind energy is about two times greater than the variations of kinetic energy. So, for \( t=75 \, \text{[s]} \) we have obtained the following values: (35), (40), (41).

The process can be represented as in Fig. 4

\[
\omega \, \text{[rad/s]} \tag{42}
\]

**Remark 3**: It can observe that at \( t=50 \, \text{[s]} \) the system reach \( \omega_{\text{OPTIM-50}} = 541.56 \, \text{[rad/s]} \) and based on this remark we can prescribe the new value for the PMSG load and it isn’t necessary to wait until \( t=75 \, \text{[s]} \).

In the same way, for \( t=110 \, \text{[s]} \), the results are:

\[
\omega_{\text{OPTIM-110}} = 536.31 \, \text{[rad/s]} \tag{43}
\]

\[
P_{\text{PMSG-P-110}} = 12650 \, \text{[W]} \tag{42}
\]

\[
R_{\text{PMSG-REQ-110}} = 238.61 \, \text{[\Omega]} \tag{43}
\]

The process is represented in Fig. 5

\[
\omega \, \text{[rad/s]} \tag{42}
\]
Remark 4: The control of the PMSG load has a dead-time of 35 [s], because the optimal load can be done only after processing the data from interval $\Delta t = 75+110$ [s]. The time variation of MAS with (REAL) and without (IDEAL) considering the dead-time is presented in Fig. 6.

The control algorithm

By measuring the wind speed, the optimal MAS can be calculated. Comparing the optimal MAS with the current MAS, the required power for the PMSG and, consequently the optimum DC link current are obtained.

The algorithm is presented below:
1. measure of wind speed and calculation of $\omega_{OPTIM,t}$
2. measure MAS of PMSG and calculation the real kinetic energy
3. estimation of the captured wind energy
4. estimation of the kinetic energy, necessary to lead the system at $\omega_{OPTIM,t}$
5. estimation of the energy from the PMSG to lead the system to MAS
6. calculation of medium PMSG power, corresponding to the energy estimated at 5.
7. calculation of the PMSG load from the power estimated at 6.
8. calculation of the PMSG power.

The value of the optimum DC link current is achieved by an appropriate control of the switches of the power electronic converter (Fig.7.)

The wind speed is measured using an anemometer. The optimum DC link current is calculated and, after that, the converter is controlled with the output value of the regulator $R$.

3.2. The relationship between the wind speed and the DC link current

The relationship is presented below:

$$U_{cc} \cdot I_{cc} = k_{cc} \cdot V^3$$

$$I_{cc} = k_1 \cdot V^3$$  \hspace{1cm} (44) \hspace{1cm} (45)

Where $k_1 = WT+PMSG$ constant and $V =$ wind speed.

The constant $k_1$ is obtained from $I_{cc,OPTIM}$ for the values obtained at $t=75$[s].

The DC link current is obtained from the wind speed, using the relationship:

$$I_{cc} = 4.0562 \times 10^{-3} \cdot V_{ECH}^3$$  \hspace{1cm} (46)

IV. CONCLUSIONS

The simulations presented in this paper have described the time evolution of the significant variables of process: current, speed, power, imposing the PMSG load. The best results are obtained by imposing the optimal value of load current, $I_{cc,OPTIM}$. By knowing the optimal value of the load current, the PMSG load can be adjusted so that the PMSG operates at the maximum energy. The speed variation of wind speed in time and the inertia value are two fundamental elements upon which the MPP operation. By prescribing the optimal DC link current, $I_{cc}$, from intermediate circuit of the converter, a simple and useful adjustment WT PMSG system can be achieved. Operation control method in the optimal energy of WT is based on the knowing of the $I_{cc}$ value, which is determined by wind speed and momentary mechanical angular speed, MAS. By analyzing several cases were able to establish basic parameters leading to an optimal operation. By measuring the wind speed, the MAS, and calculation of the optimal load current, the operation in the energetically optimal region can be performed. The control algorithm based on energy balance measurements made by MAS and electrical energy, has been validated by simulations.

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