Propagative interactions of guided waves in structured plates

Olivier Bareille, Christophe Droz, Jean-Pierre Lainé, and Mohamed N. Ichchou

Abstract—High-order waves’ propagation with low spatial attenuation in broadband frequency range is here investigated as an alternative to some structural health monitoring techniques based on first-order wave propagation. It may encompass some of the drawbacks encountered when dealing with boundary conditions in 2D-waveguides or provide accurate wave-based inspection techniques for heterogeneous or composite beams. The wave propagation and scattering in a structured composite component is studied using time-response analysis and compared to the Wave Finite Element Method’s predictions. These waves are generated by pulse excitations of the medium. The results are based on the propagation of high-order waves in a sandwich plate made of transverse isotropic honeycomb core surrounded by fiber-reinforced skins.

I. INTRODUCTION

For the last decades, numerical and experimental methods were extensively investigated to provide efficient and accurate Structural Health Monitoring (SHM) and Non Destructive Evaluation (NDE) techniques for automotive and aerospace industry [1]. Conventional NDT techniques using waves propagation are based on local ultrasonic phenomena. At high frequencies, a dispersive medium can induce a fast spatial decay of the waves amplitude. As a consequence, it becomes difficult to use the amplitude from reflected or transmitted signals for detection purpose and let alone for any monitoring use. However, for given structural configurations, waves like Lamb waves are prone to travel relatively long distances within the plates thickness. Such a waveguide architecture is often present in structures (ribbed plates, pipes, beam-like elongated components...).

Lamb’s waves have been intensively studied and applications have been proposed for structural health monitoring, based on guided waves interactions [2]–[4]. Wilcox & al. [5] have used various experimental instruments in order to generate and measure such waves. Electromechanical devices as well as piezoelectric sensor-actuator elements have been designed and field-tested [6]. Depending on the application, some types of waves are preferred to others [7]. As a consequence, even if the A0 Lamb’s wave exhibit a large out-of-plane surface displacement, modes with a much smaller amplitude like the S0 or even the in-plane motion of the SH0 mode can be preferably used and therefore the ability of the transducer pairs to generate and to measure them must be assessed [5].

When interacting with singularities, scattering and wave-mode conversions offer as many clues to the presence of these latter. However, based on numerical simulations, sensitivity analysis are still needed and reliable model for interpretation still need to be established to estimation both precise location and level of severity.

For such a purpose, the Wave Finite Element Method (WFEM) is particularly adapted. It uses Bloch’s theorem [6] to provide significant reduction of the modelling effort, since it combines the Periodic Structures Theory (PST) with commercial finite element packages [9], [10]. Therefore, wave dispersion characteristics of a waveguide whose cross-section is modelled with FEM can be derived by solving a small quadratic eigenvalue problem [11].

A. Stiffened panel case study

Guided waves occurrence is not limited to structures whose shape exhibits a significant elongation in one direction. Indeed, guided waves can be generated in other structures with specific inner-structure settings. This can be illustrated by the following results obtained for a stiffened panel under harmonic point excitation.

The structure of the panel is depicted in figure 1. The finite element model and geometry of the ribbed panel

![Fig. 1. The finite element model and geometry of the ribbed panel](image-url)
As a concluding remark here and a preamble to what is going to be developed in the following sections, the use of structural waves can be motivated by there range of frequency and by the properties of their spatial distribution. Even for frequencies far below the Rayleigh-Lamb wave applications’ ones (\( i \) 100 kHz), in the mid-frequency range, longer-wavelength long-distance control procedures can be foreseen. As a consequence, in the next sections, the propagative waves will be the physical phenomenon that will be considered to support the structural health analysis. However, their sensitivity to local singularities still need to be tested, before they can prove to help identifying local mechanical settings and properties.

B. Higher-order waves

Although wave-based methods are extensively employed in the offshore and aerospace industries for inspecting defects and cracks in 1D and 2D waveguides, these approaches are often involving first-order waves, such as the flexural, or Lamb waves for beams, plates, laminated or sandwich panels, and torsional waves in pipelines inspection [12]. However, when composite or large-scaled 1D waveguides are considered, first-order waves can be prone to coupling effects, or unaffected by localized defects. In this case higher-order, or localized waves may be used instead. In this paper, the propagation of high-order plane waves in a sandwich plate made of transverse isotropic honeycomb core surrounded by fiber-reinforced skins is investigated. These waves are created under pulse sinusoidal excitation. The wave scattering effects are studied using time-response analysis in the composite plate and compared to the WFEM predictions. Furthermore, these high-order waves have low spatial attenuation in broadband frequency range and can be used as an alternative to several SHM techniques based on first-order wave propagation. Some of the drawbacks could hence be circumvented, encountered when dealing with boundary conditions in 2D-waveguides or provide accurate wave-based inspection techniques for heterogeneous or composite beams.

II. Wave finite element method (WFEM)

A waveguide is considered as a straight elastic structure made of N of identical substructures of same length d, connected along the direction x. The state vector is described in figure 3. Nodal displacements and forces are denoted \( q \) and \( f \), where the subscripts ‘L’ and ‘R’ describe the cell’s left and right faces. Both edges have the same number \( n \) of degrees of freedom. Mesh compatibility is assumed between the cells. The governing equation in a cell at frequency \( \omega \) is written:

\[
(-\omega^2 M + K)q = f
\]  

where \( M, K \) are the mass and complex stiffness matrices, respectively. A dynamic condensation of the inner DOFs can be required if the structure is periodic. The governing equation can be written by reordering the DOFs:

\[
\begin{bmatrix}
K_{LL} & K_{LR} \\
K_{RL} & K_{RR}
\end{bmatrix}
-\omega^2
\begin{bmatrix}
M_{LL} & M_{LR} \\
M_{RL} & M_{RR}
\end{bmatrix}
\begin{bmatrix}
q_L \\
q_R
\end{bmatrix}
= \begin{bmatrix}
f_L \\
f_R
\end{bmatrix}
\]

(2)

where \( M_{ii} \) and \( K_{ii} \) are symmetric, \( M_{RL} = M_{LR} \) and \( K_{RL} = K_{LR} \), \( \lambda = e^{-jkd} \) is the propagation constant, describing wave propagation over the cell length \( d \) and \( k \) is the associated wavenumber, considering force equilibrium

\[
\lambda f_L + f_R = 0
\]  

(3)

in a cell and Bloch’s theorem:

\[
q_R = \lambda q_L
\]  

(4)

into Eq. (2), it yields the following spectral eigenproblem:

\[
S(\lambda, \omega) = (\lambda D_{LL} + (D_{LL} + D_{RR}) + \frac{1}{\lambda} D_{RL} + )\Phi = 0
\]  

(5)

where the solutions \( \Phi \) stand for the wave shape associated with the displacements \( q_L \) of the waveguide’s cell. In damped waveguides, complex wavenumbers are associated to decaying waves. Defining the state vector : \( \Phi = [(\Phi_q)^t, (\Phi_f)^t]^t \), the spectral problem can be written using the symplectic transfer matrix \( T \):

\[
T\Phi = \lambda \begin{bmatrix}
\Phi_q \\
\Phi_f
\end{bmatrix}
\]  

(6)

with

\[
T = \begin{bmatrix}
D_{LL}^{-1} & D_{LR}^{-1} \\
D_{RL} - D_{RR}D_{LR}^{-1}D_{LL} & -D_{RR}D_{LR}^{-1}
\end{bmatrix}
\]

(7)

Here, the waves associated with positive wavenumber are travelling in the positive x-direction and the negative wavenumbers describe propagation in the negative x-direction. The dynamical behaviour of the global system can be expressed by expanding amplitudes of incident and reflected waves on a basis of eigenvectors. If the structure is undamped,
solutions are divided into propagative waves, whose wavenumbers are real, and evanescent waves for which wavenumbers are imaginary. In dissipative case, complex wavenumbers are associated to decaying waves.

### III. Dispersion Characteristics of a Sandwich Plate

#### A. Description of the Composite Waveguide

The rectangular sandwich waveguide is composed of a 8 mm thick homogenised honeycomb core surrounded by 1 mm thick fiber-reinforced skins. The 400 mm width cross-section is modelled using 360 linear block elements having 8-nodes and 3 degrees of freedom (DOF) per node. The waveguide is described in figure 4, a structural loss factor \( \eta = 0.01 \) is assumed and a detailed description of the materials is given in tables I and II.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg.m(^{-1}))</th>
<th>Young Modulus (Pa)</th>
<th>Shear Modulus (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nomex</td>
<td>24</td>
<td>( E_x = 5 \times 10^9 )</td>
<td>( G_{xy} = 1 \times 10^7 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( E_y = 5 \times 10^9 )</td>
<td>( G_{xz} = 10.13 \times 10^6 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( E_z = 46.6 \times 10^6 )</td>
<td>( G_{yz} = 10.13 \times 10^6 )</td>
</tr>
</tbody>
</table>

**TABLE I**

Material Properties of Honeycomb Core

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg.m(^{-1}))</th>
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</tr>
</thead>
<tbody>
<tr>
<td>TC skin</td>
<td>1451</td>
<td>( E_x = 81 \times 10^9 )</td>
<td>( G_{xy} = 2.5 \times 10^7 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( E_y = 81 \times 10^9 )</td>
<td>( G_{xz} = 2.8 \times 10^8 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( E_z = 3.35 \times 10^9 )</td>
<td>( G_{yz} = 2.8 \times 10^9 )</td>
</tr>
</tbody>
</table>

**TABLE II**

Material Properties of Fiber-Reinforced Skins

Fig. 4. Finite element model of sandwich plate involving finite width.

#### B. Propagating Waves and Shapes

The wavenumbers associated with the propagating waves in the sandwich waveguide are shown in figure 5. The continuous lines (—) describe first-order waves while dashed lines (— - ) represent high-order propagating waves, associated with deformed cross-sections. It can be noticed that numerous high-order waves are propagating in this structure, in addition to the four first-order waves (transverse and in-plane flexural, torsional and longitudinal waves). The cross-sectional deformed shapes associated with these first-order waves are shown in figure 6. In the considered structure, high-order waves are associated with sinusoidal deformation of the waveguide’s cross-section. Their shapes are described in figure 7. The spatial attenuations of the propagating waves in the frequency range \([0, 4000]\) Hz are shown in figure 8. The wave amplitudes are given after a one meter propagation in the main direction. Although high-order waves share the same asymptotic group velocity of the first-order flexural and torsional waves, their spatial attenuations exhibit different behaviour close to each of their cut-on frequencies.

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</tr>
</tbody>
</table>

**Fig. 5.** Real part of the wavenumbers associated with propagating, positive-going waves.

**Fig. 6.** Deformed shapes associated with the first-order propagating waves.

**Fig. 7.** Deformed shapes associated with high-order propagating waves.

**Fig. 8.** Amplitudes of the propagating waves in the sandwich waveguide after 1 meter propagation.

### IV. Time Analysis Using Wave Appropriation

This work is concerned with the propagation of the aforementioned high-order waves in a sandwich plate of finite dimensions. Therefore, the actuation of the waves described in figure 7 is proposed using localized vertical displacements. The shape appropriation is shown in figure 9 for the 4\(^{th}\) order flexural wave.
The transient response under pulse train excitation (see figure 10) is determined using time-explicit simulation. The frequency spectrum of the pulse is described in figure 11. A reduced dispersion of the pulse train can be obtained by narrowing the frequency spectrum bandwidth. It can be done by increasing the number of periods in the pulse train. In figure 12, the time response of the waveguide is described under a 2\textsuperscript{nd}-order wave at 1k Hz. Noteworthy, the wave pulse propagates without coupling effects and a slight dispersion. It can be explained since the frequency spectrum involves different group velocities for a given wave. Therefore, it seems advantageous to generate waves at higher frequencies. Similarly, the propagation of the 4\textsuperscript{th}-order wave is shown for a 2k Hz pulse involving 8 periods is shown in figure 13.

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### REFERENCES


