Urban risk assessment: fragility functions for masonry buildings

D. Ottonelli, S. Cattari and S. Lagomarsino

Abstract—The seismic risk analysis of buildings at urban scale requires the effective classification of the exposure and the definition of the vulnerability through reliable fragility curves. Fragility curves give the probability of attainment of certain damage measures (limit states) as a function of a proper intensity measure of the earthquake. A suitable taxonomy for ordinary masonry buildings and its use in the assets classification is proposed. With regards to vulnerability models a procedure for estimating the fragility curves, from both expert elicitation and analytical models, is described. In particular, the latter considers a rigorous evaluation of uncertainties.

Keywords—Seismic vulnerability, Risk assessment, Fragility curves, Masonry, Taxonomy.

I. INTRODUCTION

THE definition of fragility functions for masonry buildings is a hard task because we refer to a wide variety of constructions, which are characterized by very different types of masonry and structural systems, moving through historical periods and geographical areas.

As regards the first point, masonry is a composite material and the mechanical properties are related not only to those of the constituents, blocks (stone, solid clay bricks, adobe, etc.) and mortar (mud, lime, hydraulic lime, cement), but also to the dimensions and shape of the blocks, the interlocking in the external leaves and the transversal connection through thickness.

With reference to the structural systems, ancient constructions, but also recent vernacular ones, are very different from engineered masonry buildings, such as confined or reinforced masonry. The former were built by an empirical approach and are usually vulnerable, first of all to local mechanisms (out-of plane behavior); however, in high seismic areas specific details were adopted to prevent from damage (metallic tie rods, timber belts, buttresses, connections of horizontal diaphragms to masonry walls, etc.). The latter have been specifically conceived to withstand the earthquake, after a detailed damage observation, as in the case of confined masonry (widely adopted in South American countries), or on the base of modeling and capacity design criteria, as in the case of unreinforced masonry building (with RC, reinforced concrete, ring beams at floor level) or reinforced masonry.

Among the masonry building may also be considered the mixed structures, such as the traditional mixed masonry-timber buildings or the rather modern mixed masonry-RC buildings. The former may have different configurations: a) timber reinforced masonry buildings, with horizontal timber ties at various levels and connected through thickness (e.g. in the Balkan, Greek and Turkish area); b) timber-framed masonry buildings (e.g. frontal walls of Pombaline buildings in Portugal, or smaller building with main bearing walls confined and braced with timber elements, all over the world); c) buildings with masonry walls at the lower stories and timber frames at the upper ones. Besides confined masonry, the spread of RC technology in the first half of 20th century has caused the birth of different types of mixed masonry-RC buildings, results of functional choices and often quite vulnerable: a) masonry perimeter walls and RC interior frames; b) raising of masonry buildings with RC framed structures.

Another important distinction is between ordinary and monumental masonry buildings. The latter category collects special type of assets, from the morphological point of view, such as: churches, mosques, towers, minarets, fortresses, etc.; they have a specific seismic behavior and, usually, a higher vulnerability, as testified by the last seismic events. Models and fragility functions defined for ordinary masonry buildings can be also used for monumental palaces, but in addition it is required an additional vulnerability assessment of some specific elements, if present (loggias, cloisters, colonnades, wide halls with double height, etc.).

This paper is focused on ordinary masonry buildings. In particular mechanical models and fragility functions are proposed for ordinary unreinforced masonry buildings. However, the general framework of the procedure outlined in §III, in terms of key assumptions, uncertainties treatment and modeling issues, can be adopted also for the derivation of fragility functions of other masonry buildings typologies.

In Table 1 the main features that are useful for the taxonomy of masonry buildings are listed. Each building is described by a string of codes, separated by slashes and hyphens. Slashes mark the main categories of the taxonomy: FRM – Force Resisting Mechanism; FRMM – Force Resisting Mechanism Material; P – Plan; E – Elevation; CO – Cladding & Openings; DM – Detailing & Maintenance; FS – Floor
System; RS – Roof System; HL – Height Level; CL – Code Level. Within each category, the list of possible options is defined by proper acronyms; a more detailed classification and sub-classification (in square brackets, Table 1) is related to some of the category options and can be indicated in the taxonomy by separating codes by hyphens.

Table 1 proposal of taxonomy for Masonry Buildings

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>CLASSIFICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing Walls (BW)</td>
<td>Out of plane (OP); In plane (IP)</td>
</tr>
<tr>
<td>Unreinforced Masonry (URM)</td>
<td>Blocks: Adobe (A); Fired brick (FB); Soft Stone (SS); Hard Stone (HS)</td>
</tr>
<tr>
<td>Reinforced Masonry (RM)</td>
<td>Mortar: Lime mortar (LM); Cement mortar (CM); Mud mortar (MM); Hydraulic mortar (HM)</td>
</tr>
<tr>
<td>Confined Masonry (CM)</td>
<td>Timber: Confined and braced masonry panels (TC); Horizontal timber tie (TT)</td>
</tr>
<tr>
<td>Timber-framed Masonry (TM)</td>
<td>Concrete and reinforcement: [Average Strength (20-50 MPa)/ASC], Low Strength (&lt;20 MPa)/LSC]; [Vertical Reinforcement Bars (RVB), Vertical and Horizontal Reinforcement Bars (RVBR)]</td>
</tr>
</tbody>
</table>

In the case of masonry buildings the FRM is always the Bearing Walls system (BW), which can present very different seismic behavior depending on geometry and constructive details. Usually reference is made to Out-of-Plane (OP) and In-Plane (IP) mechanism, depending on the connections and distance between masonry walls, as well as on the stiffness of horizontal diaphragms. If a global seismic (box-type) behavior can be assumed, a sub-classification is possible: each single wall may be analyzed by an equivalent frame model (EF) or by simplified models that assume the hypotheses of strong (SSWP) or weak (WSSP) spandrels. The choice of the most reliable model depends on available as-built information.

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The configuration of the building Plan (P) is very important for the seismic vulnerability, both with reference to the regularity (R, IR) and to the possible interaction with other buildings (Isolated – I – or Aggregated in urban blocks – A). This information is useful to address the most probable collapse mechanisms (BW classification).

Information on the regularity in Elevation (E) may help in the definition of the behavior factor and the ductility, due to the possible different localization of the weak story.

The role of non-structural elements is almost negligible in masonry buildings, but it is important to know the regular distribution and percentage of openings (CO). A regular distribution (RO) may promote the WSSP behavior, which is characterized by a higher displacement capacity but a lower strength than the SSWP case. Moreover, a High percentage of openings (H%) at the base story, typical in the case of shops, may produce a weak story mechanism, which has a low displacement capacity.

Another important category, in particular in the case of URM buildings, is the quality of constructive details and the state of maintenance, which is an essential prerequisite in order to exploit the former aspect (DM). The attribution of High Quality Details (HQU) must consider the adherence to the rules of the art, which altogether define a local code of practice referred to different scales of the construction: the masonry (way to assure interlocking and transversal connection), the wall (distribution of openings, lintels, etc.) and the global construction (wall-wall and wall-horizontal diaphragms connections). The systematic presence of effective tie rods (WT) or ring beams (WRB) may prevent from out-of-plane mechanisms and increase the strength and ductility of spandrels, for the in-plane behavior; it is worth noting that RC ring beams drive the seismic response to weak story mechanism (SSWP behavior), while tie rods increase the ductility of uniform mechanisms (WSSP behavior).

The Floor System (FS) influences the seismic behavior, with reference both to its mass (which increases the horizontal seismic actions) and its stiffness (which allows a certain degree of redistribution of the horizontal seismic actions between the vertical walls). A rough categorization is obtained by distinguishing between Rigid (R) and Flexible (F); the attribution has to consider not only the stiffness but also the effectiveness of the connection with vertical walls. A more detailed classification can consider also the material and configuration (i.e. the presence of masonry vaults can also induce horizontal thrusts).

The Height Level (HL) is very important because it influences very much the seismic vulnerability and is always available or very easily detectable. The possible categories (L, M, H and Ta) must be redefined, in terms of number of stories,
for masonry buildings, because they are on average lower than RC or steel buildings.

Finally, the Code Level (CL) category is very important and must be properly defined in the case of masonry buildings, which are usually old and not seismically designed (PC); in this case, it is useful to estimate the local seismic culture, which is high (HAC) in areas frequently affected by earthquakes. For modern buildings, designed by considering a seismic code (LC, MC and HC), the categories should mainly consider the seismic hazard used for the design, taking also into account the accuracy of the code provisions.

The vulnerability assessment at territorial scale requires to group the buildings that have a similar seismic behavior in order to evaluate the damage and losses of the built environment due to a given hazard assessment. To this aim, the proposed taxonomy cannot be directly used, because available information is always incomplete and, anyway, a too very detailed subdivision of the building stock considered in the risk analysis might be useless and difficult to be managed.

Depending on the available data and after a preliminary study of the characteristics of the built environment in the urban area under investigation, the first step of the vulnerability assessment is to proceed to a proper classification of buildings. To this aim, among the available information, the parameters that mostly affect the seismic behavior must be singled out. Each vulnerability class, which can be synthetically named by a number or a short acronym, is clearly identified by a precise taxonomy, that is a list of category and related classification information. Missing information in the taxonomy means that no data are available to better describe the buildings, so fragility functions must represent the average vulnerability of a large set of configurations. On the contrary, if some parameter is excluded, all other options should be listed in the taxonomy.

Fragility functions must be defined, according to suggestion of section §IV, for each building class. It is worth noting that the dispersion is higher when few building classes are used, each one including constructions characterized by quite different behavior; on the contrary, a too much detailed classification may lead to the definition of classes with quite similar fragility functions, but with a lower dispersion.

As an example, in the case of a risk analysis at regional scale, when little information is available, the following tags of the taxonomy could describe a possible classification:

\[ \text{Class 1: } /BW/URM-FB-HM/R/R/RO/HM/R/P/M/PC-MAC/ \]

If the analysis is focused on a urban district, with a small number of buildings, it is possible to limit the possible options, after a quick sample check survey, and split the classes proposed above, on the base of: quality of seismic design and construction details, materials of floor and roof system, etc.

II. REVIEW OF EXISTING FRAGILITY FUNCTIONS

Many fragility functions have been developed and can be taken from the literature for the risk analysis of masonry buildings [1]-[10]. They have been derived according to different approaches, which can be traced back to the classification of §III.B: 1) empirical (e.g. [11]-[13]); 2) expert elicitation based (e.g. [14]); 3) analytical, based on nonlinear static approaches through simplified (e.g. [2], [15]-[24]) and detailed models (e.g. [25]) or based on linear dynamic approaches (e.g. [26], [27]); 4) hybrid methods (e.g. [28]).

Many fragility functions have been obtained from observed damage after the occurrence of an earthquake; these data are valuable, because they are directly correlated to the actual seismic behavior of buildings and can be very useful for validation of analytical methods and calibration of hybrid fragility functions. However, empirical fragility functions are strongly influenced by the reliability of the damage assessment, which is often made by a quick survey aimed to other scopes, as the building tagging for use and occupancy.

Once in the study area masonry building typologies have been analyzed and building classes defined, it is necessary to derive the appropriate fragility functions. To this end, for each class, fragility functions taken from different authors may be used and properly combined, but attention must be paid because these functions could be biased due to some parameters or aspects.

First of all, a crucial factor is the choice of the seismic intensity measure. Empirical data are usually referred to macroseismic intensity, which is not an instrumental measure but is based on a subjective evaluation. This approach is suitable when the aim of the risk analysis is to draw a comparative scenario, probably useful to plan mitigation strategies; for an accurate loss estimation, however, it is necessary to convert macroseismic intensity into an instrumental intensity measure, and this step introduces important approximation and normally huge uncertainties. On the contrary, if empirical fragility functions are given in terms of Peak Ground Acceleration (PGA), it is worth noting that this parameter is directly related to the spectral characteristics of the input motion of the specific seismic event. In these cases, the correlation between intensity and damage should present a low dispersion, which has to be increased before using those functions.

Another difficult task is the definition of consequences that are evaluated by the fragility functions. Usually Damage States (DS) are considered, which are referred to physical damage to structural and non-structural elements, but fragility functions can be also drawn in terms of a Damage Index (DI), related the cost of repair, or of some Performance Indicators (PIs), which are related to the conditions of use (operational, occupancy, life safety). All the above-mentioned effects (except DI) are discrete states and are defined by a qualitative judgment (in case of observational functions) or by a correlation with some structural parameter, as the interstory drift (in case of analytical based functions).

Finally, it is worth noting that the characteristics of masonry buildings are dependent from the local seismic culture and the available materials in the area; as an example, the apparently detailed description “irregular stone masonry with lime mortar” may correspond to very different seismic capacities, if it is assigned to buildings in different countries. Thus, the
extrapolation of empirical fragility functions for traditional masonry buildings to other geographic areas is questionable.

In conclusion, the use of existing fragility functions has to be made carefully. In order to increase the reliability of the results, it is suggested to combine a significant number of fragility functions, obtained from different authors and with different methods, assigning to each one a proper subjective probability, related to the reliability of the source and the fitting with the characteristics of the building class under investigation, in order to obtain a weighted fragility function. Depending on the availability and reliability of fragility functions, the building classification should be more or less detailed. An excessive splitting of the built environment into detailed classes, with associated low dispersed fragility functions, turns out to be specious if their reliability is not robust; in these cases it is better to reduce the number of buildings classes and ascribe to each one a more reliable fragility function, even if defined by a bigger dispersion.

In the context of analytical based on nonlinear static approaches, this chapter proposes a procedure to derive fragility functions for masonry buildings, once a building class is defined by tagging the various categories of the taxonomy (Table 1). The general framework of the method, the probabilistic key assumptions and the modeling bases are treated in §III, while some operative recommendations for the different possible approaches are given in §IV. The development of tailored fragility functions is the suggested way to improve the reliability of the vulnerability and risk analysis.

III. UNCERTAINTIES AND MODELING ISSUES

The fragility function gives the probability that a generic Limit State (LS) is reached given a value \( im \) of the Intensity Measure IM:

\[
p_{LS}(im) = P(d > D_{LS}(im)) = P(im_{LS} < im) = \Phi \left( \frac{\log (im/im_{LS})}{\beta_{LS}} \right)
\]

where: \( d \) is a displacement representative of the building seismic behavior, \( D_{LS} \) is its Limit State threshold, \( IM_{LS} \) is the median value of the lognormal distribution of the intensity measure \( im_{LS} \) that produces the LS threshold of the and \( \beta_{LS} \) is the dispersion.

A fragility function is thus defined by two parameters: IMLS and \( \beta_{LS} \). The median intensity IMLS can be obtained from the statistical analysis of data from damage observation after earthquakes (empirical methods) or by a mechanical model (analytical methods), that is representative of the average seismic behavior of buildings of that particular class.

The dispersion \( \beta_{LS} \) depends on different contributions, related to: a) the uncertainties in the seismic demand (epistemic \( \beta_{D} \), for the derivation of the hazard curve, and intrinsic \( \beta_{D} \), in the variability of the seismic input described only by the value of IM); b) the uncertain definition of the Limit State threshold (\( \beta_{T} \)); c) the variability of the capacity (\( \beta_{C} \)) of buildings that belong to the considered vulnerability class (which collects buildings of different behavior, even if characterized by the same taxonomy tags). As all the above contributions can be assumed statistically independent, the dispersion is given by:

\[
\beta_{LS} = \sqrt{\beta_{D}^2 + \beta_{T}^2 + \beta_{C}^2}
\]

In case of analytical methods each contribution can be computed, while for empirical methods \( IM_{LS} \) is directly evaluated from the damage distribution of observed data, which includes all of them; however, in this case, it is necessary to verify if the dispersion has to be increased, because empirical data are not fully representative, in terms of masonry typology (\( \beta_{C} \)) or characteristics of the input motion (\( \beta_{D} \)).

The following sub-sections describe the main aspects related to the derivation of fragility functions for masonry buildings from analytical methods, based on nonlinear static approaches.

A. Capacity and demand by nonlinear static analysis

The seismic vulnerability of building is described by its capacity curve that gives the acceleration \( A \) of an equivalent nonlinear single-degree-of-freedom system, as a function of its displacement \( D \). The capacity curve can be obtained by a proper conversion of the pushover curve, obtained by a nonlinear static analysis of a multi-degrees-of-freedom model of the structure, or through simplified analytical models. In the latter case the capacity is usually described by a bilinear curve, without hardening for masonry buildings.

The seismic demand is expressed by an Acceleration-Displacement Response Spectrum (ADRS), which gives the spectral acceleration \( S_a \) as a function of the spectral displacement \( S_d \), for a damping coefficient \( \xi_d =5\% \), considered valid in the initial elastic range. Usually in hazard analysis the spectral shape is assumed constant with the annual rate of exceeding, which is given by the hazard curve as a function of a proper IM of the ground motions.

The evaluation of the displacement demand for a given value \( im \) of the IM can be obtained through various methods, like the N2-Method originally proposed by Fajfar [29], the Capacity Spectrum Method [30], the Displacement-Based Method [16], Coefficient Method [31], [32], the MADRS Method [33]. They all consider, under different approaches, the reduction of the seismic demand in the nonlinear phase of the building response. These methods look for the intersection of the capacity with the properly reduced demand, by using either acceleration/dispacement or displacement/period as coordinates (\( S_T = S_d T^2/4\pi^2 \)).

For the evaluation of fragility functions it is necessary to get the value \( IM_{LS} \) of the IM that produces any LS threshold. To this end the use of over-damped spectra [30] is very effective, once these thresholds \( D_{LS} \) have been fixed on the capacity curve (§III.C) and the corresponding equivalent viscous damping \( \xi_{LS} \) is evaluated, which also takes into account the hysteretic contribution. It results:

\[
IM_{LS} = \frac{D_{LS}}{S_d(\xi_{LS})\eta(\xi_{LS})}
\]
where: \( S_d \) is the displacement response spectrum, normalized to IM, \( T_{LS} \) is the linear equivalent period corresponding to LS:

\[
T_{LS} = 2\pi \sqrt{\frac{\zeta_{LS}}{\alpha(\zeta_{LS})}}
\]

and \( \eta(\xi_{LS}) \) is the damping correction factor [34]:

\[
\eta(\xi_{LS}) = \frac{10}{5+\xi_{LS}}
\]

It is worth noting that overdamped spectrum is obtained simply multiplying by \( \eta(\xi_{LS}) \) in the range of typical periods for buildings, while for very low and high periods the effect of damping tends to vanish. Figure 1 shows the procedure, considering LS on the capacity curve and the identification of \( IM_{LS} \), using PGA as IM and a typical response spectrum shape.

\[ \begin{align*}
I_{LS} & = S_d(T) \\
\beta_{LS} & = \frac{\alpha(T)}{\alpha(0)} \\
\end{align*} \]

where: \( \alpha(T) \) is the spectral acceleration for a given value of the period of vibration \( T \), and \( \beta_{LS} \) is the ratio of the spectral acceleration at LS to its value at zero period.

**B. Identification of proper Intensity Measures**

The vulnerability assessment, embodied by the application of fragility functions, is one of the steps of the seismic risk analysis. The identification of the proper Intensity Measure (IM) comes out from different constraints, which are first of all related to the adopted hazard model, to the typology of the exposed asset but also to the availability of data and fragility functions for all different exposed assets.

Empirical fragility functions are usually expressed in terms of the macroseismic intensity \( I \) (defined according to the different Macroseismic Scales: EMS, MCS, MM), which can be regarded as an empirical IM. The macroseismic intensity already contains implicitly the vulnerability, because it is defined on the basis of the damage observation; in order to overcome this gap, modern macroseismic scales, such as EMS, assign the intensity taking into account a detailed building types classification. The accuracy of the risk analysis results is then linked to the reliability of the hazard assessment, if an empirical IM is used.

Analytical based or hybrid fragility functions are, on the contrary, related to instrumental IMs, which are related to parameters of the ground motion (PGA, PGV, PGD) or of the structural response of an elastic SDOF system (spectral acceleration \( S_a \) or spectral displacement \( S_d \), for a given value of the period of vibration \( T \)). Sometimes, integral IMs can be useful, which consider a specific integration of a motion parameter (Arias Intensity \( I_A \) or of a spectral value (Housner Intensity \( I_H \)) [35].

Correlation is necessary when hazard and vulnerability assessments are made by using different IMs or one wants to calibrate analytical fragility functions (related to a detailed building classification) by available empirical fragility functions (referred to wider classes of buildings). Anyhow, the use of correlation always increases the uncertainties of the results (dispersion \( \beta_{LS} \) of the fragility function).

Similarly to what was said on different types of fragility functions (empirical, expert elicitation, analytical based and hybrid), for the identification of proper IMs it is worth noting that empirical ones give results coarse but correct on average, while instrumental IMs allow to better take into account a detailed taxonomy, in the definition of building classes, and the local site effects, but, when these fragility functions are used, it is necessary to pay attention to the characteristics of the input motion that was considered for their derivation.

The seismic performance of a masonry building cannot be described by only one IM but, at least, the response spectra shape should be known. If a vector-valued hazard assessment is available [36], more than one IM could be used and vector-valued fragility functions derived (e.g. [27]). If already available fragility functions are used, it is better to refer to the spectral value for the period compatible with the specific Limit State threshold (acceleration \( S_a(T) \) and displacement \( S_d(T) \) response spectra are linked by the period of vibration \( T \), so the two IMs are equivalent). In this case the dispersion \( \beta_{LS} \) of the fragility function is mainly due to the variability of the capacity of buildings in the class.

Most of available fragility functions are in terms of PGA; in this case, if the difference between the spectral shapes of the input motion obtained by the hazard assessment and that used for deriving the fragility function is known, it is possible to properly tune the last one. Otherwise, the use of PGA as IM implies a wider dispersion \( \beta_{LS} \) of the fragility function, due to the uncertainty in the spectral shape.

As masonry buildings are usually not flexible, PGD or spectral values for long periods (\( T > 1 \) s) are not significant, except for some types of monumental structures (churches, slender towers) or for the verification of local mechanisms.

With reference to local site amplification, spectral values are better correlated with vulnerability, because they take into account the modification of the seismic input for the significant periods. If PGA is used, fragility functions should be tuned by considering a mean ratio between the spectral values on local site and stiff soil conditions, for the relevant periods of the buildings, or a greater value of the dispersion should be used, in order to consider the increased uncertainty due to the spectral demand (\( \beta_{I_H} \)).

In case of using empirical IM (macroseismic intensity), it is not correct to include local site amplification in the hazard curve, because this phenomena affects buildings depending on their dynamic properties; a possible solution is to modify the empirical fragility function, so considering it as representative of the vulnerability of a particular class of buildings on a
specific soil type [14].

C. Definition of Damage States and Performance Levels

In seismic risk analysis the scenario of the built environment is expressed in terms of Damage States (DS), which are a discrete qualitative description of the overall damage in structural and non-structural elements of the building. Usually five damage states are considered: DS1 slight, DS2 moderate, DS3 extensive, DS4 near collapse and DS5 collapse.

Empirical methods describe the DS through a qualitative damage observation, on the basis of distribution and severity of cracks, according to specific forms and sketches; to this end, modern macroseismic scales can be a good reference (e.g. EMS98 [37]).

In the case of analytical methods, if a detailed numerical model of the building is available, the damage in each structural element is obtained through static or dynamic nonlinear analysis and a sort of virtual damage state attribution could be made. However, it is worth noting that numerical models give continuum damage variables and identification of discrete DS is not an easy task. As an example, [38] have proposed a multi-scale approach for masonry buildings that defines Limit States (LS) on the capacity curve by checking (i) the spread of damage in masonry elements (piers and spandrels), (ii) the interstory drift in masonry walls and (iii) the global behavior of the building (described by its capacity curve). LSs are the thresholds that separate various DSs (Figure 2).

Fig. 2. Example of capacity curve of a masonry building, obtained by pushover analysis on a detailed model, with the definition of LS thresholds and DS ranges.

Damage States can be related to specific performances of the building: the use and occupancy, the safety of people and the reparability (in terms of economic convenience). Usually Performance Limit States (PLS) are defined as coincident to related Damage Limit States (LS); this means the fulfillment of a performance is guaranteed if the seismic displacement demand is not beyond the corresponding LS threshold.

The above mentioned detailed mechanical based methods are used in hybrid approaches, while analytical methods adopt simplified models, which give directly the capacity curve without a detailed description of damage in the building. In these cases, LSs may be defined: a) by considering limit values of macro-parameters of the building response, on which the simplified model is based (as, for example, the interstory drift); b) by a heuristic approach, which considers that the transition from a DS to the following one usually occurs in certain positions of the capacity curve. In the latter, a possible positioning of LSs is obtained as follows (Figure 3a): LS1: \( D_L = 0.7D_y \); LS2: \( D_L = c_2 D_y \); LS3: \( D_L = (1-c_3)D_y \); LS4: \( D_L = D_u \). The position of LS2 depends on the complexity and irregularity of the building; the coefficient \( c_2 \) may vary between 1.2 and 2, being lower for simple and regular buildings. LS3 is usually closer to LS4, in particular for simple and regular buildings (0.3<\( c_3 <0.5 \)).

Equivalent viscous damping may be defined for each LS as a function of the displacement (Figure 3b), by a simple relation [16], [39], [40]:

\[
\xi_{LS} = \xi_0 + \xi_H \left[ 1 - \left( \frac{D_L}{D_y} \right) \right]
\]

where: \( \xi_0 \) is the initial damping (usually assumed equal to 5%), \( \xi_H \) is the maximum hysteretic damping and \( \xi \) is a free parameter (ranging between 0.5 and 1).

Once the seismic demand is defined (§III.B), by the spectral shape and the selection of a proper Intensity Measure to scale it, the values \( IM_{LSk} \) and the dispersions \( q_{LSk} \) (k=1...4) can be evaluated by (3) and by the procedure described in §III.D. Fragility curves are then given by (1) and shown in Figure 4a.

The DS probability distribution, for a given value of the IM, can be thus obtained from fragility functions; for k=1, 2 and 3, the discrete probabilities are given by:

\[
p_{DSk}(im) = p_{LSk}(im) - p_{LSk+1}(im) = \Phi \left( \frac{\log \left( \frac{im}{IM_{LSk}} \right)}{q_{LSk}} \right) -
\]
With regards to DS4, it is worth noting that analytical methods usually are not able to define LS5, and thus \( p_{LS5} \); this LS occurs after important local collapse mechanisms that make the mechanical model meaningless. If it is considered that DS4 is generically named “complete” damage, including both “near collapse” and “collapse” DSs, it results that \( p_{DS4} = p_{LS4} \). However, by assuming that the probability distribution of DSs is well represented by the binomial distribution, it is possible to share \( p_{LS4} \) according to the following formulas (7):

\[
p_{DS4}(im) = 0.8\left[1 - (1 - 0.14\mu_{DS4})^{0.35}\right]p_{LS4}(im) \tag{8}
\]

\[
p_{DS5}(im) = p_{LS4}(im) - p_{DS4}(im) \tag{9}
\]

In order to complete the DS distribution it is necessary to evaluate the probability that the building has “no damage” (DS0):

\[
p_{DS0}(im) = 1 - p_{LS1}(im) = 1 - \Phi\left(\frac{\log(m_{DS0})}{\mu_{LS1}}\right) \tag{10}
\]

Figure 4b shows a typical discrete damage distribution of damage states, directly obtained from fragility functions of Figure 4a for a given value IM=im.

**D. Sources of uncertainties and propagation**

In a probabilistic seismic risk analysis many uncertainties have to be taken into account; Pinto gives a general overview in chapter §II. Their propagation is considered in fragility functions through the dispersion \( \beta_{LS} \), which can be evaluated by Eq. (2). The estimation of different contributions is discussed in the following.

\( \beta_{D} \) – Uncertainty on the spectral shape of the seismic demand

The Probabilistic Seismic Hazard Analysis (PSHA) gives the occurrence of earthquakes with a proper IM through the hazard curve \( \lambda(im) \). Usually a fixed shape of Acceleration-Displacement Response Spectrum is associated, except the case of a complex Vector-Valued PSHA [36]. The normalized response spectrum \( S_d(S_t) \) scaled to the value \( im=1 \), can be defined as a stepwise function or through some analytical formulas in fixed ranges of the period \( T \) (as it is made in seismic codes).

In order to take into account the uncertainty on the spectral shape, which plays a significant role due to the large variability of possible records, it is necessary to define the response spectra \( S_{d,16}(S_t) \) and \( S_{d,84}(S_t) \), for the confidence levels 16% and 84%. They can be obtained by the selection of a large number of real digital records, compatible with the characteristics of earthquakes that give the maximum contribution to the hazard and of soil conditions; in particular, from the disaggregation of the PSHA, it is important to consider: magnitude, epicentral distance, focal depth, source mechanism. Figure 5a shows a typical example of a median response spectrum and the corresponding confidence intervals, if the Peak Ground Acceleration PGA is used as IM; Figure 5b shows the same response spectra in the case where the maximum spectral acceleration \( S_{a,max} \) is assumed as IM.

\[
\beta_{D} = 0.5\left[\log(IM_{D,84}) - \log(IM_{D,16})\right] \tag{11}
\]

This contribution to the dispersion is lower if a good IM is used. It is quite evident from Figure 5 that, at least for LS1 and LS2, \( S_{a,max} \) is better than PGA.

\( \beta_{H} \) – Epistemic uncertainty on the hazard curve
Epistemic uncertainties in the seismic sources and the attenuation laws give rise to confidence intervals, which can be summarized by the hazard curves $\lambda_{im}(\text{im})$ and $\lambda_{84}(\text{im})$, representative of the confidence levels 16% and 84%.

For each LS, it is necessary to valuate $IM_{LS}$ that corresponds to the displacement demand $D_{LS}$ on the median capacity curve of the considered building class, by using the median response spectrum $S_\alpha(S_\delta)$. The dispersion $\beta_H$ is given by:

$$\beta_H = \frac{1}{2} [\log(IM_{H,16}(\lambda(IM_{LS}))) - \log(IM_{H,84}(\lambda(IM_{LS})))]$$  \hspace{1cm} (12)

where $IM_{H,16}$ and $IM_{H,84}$ are the inverse functions of $\lambda_{im}(\text{im})$ and $\lambda_{84}(\text{im})$, respectively, and $\lambda(IM_{LS})$ is the median hazard curve.

\[ \text{Fig. 6. Influence on the spectral demand of the epistemic uncertainty on the hazard curve.} \]

Figure 6 shows an example of hazard curves, median and confidence intervals, and the corresponding response spectra; in this case for the evaluation of $\beta_H$ only the median response spectrum is used.

$\beta_C$ – Uncertainty on the capacity curve

The dispersion on the capacity curve of a masonry building is related to random variables, such as the material parameters (strength and stiffness on masonry), the geometry (effective thickness of masonry walls and vaults), the drift capacity of masonry panels or the in-plane stiffness of horizontal diaphragms, but also to epistemic model uncertainties, related for example to the assumptions in the definition of the equivalent frame or in modeling the connection between walls. Usually, if accurate as-built information is available, these uncertainties can be reduced.

This is not the case in seismic vulnerability analysis at territorial scale, when an “equivalent capacity curve” must be evaluated representative of a wide class of buildings, defined by the taxonomy through a proper list of tags. Then the above parameters have to be considered as random variables, with a dispersion compatible with the variability of the characteristics of buildings in the class.

The uncertainty propagation can be evaluated by Monte Carlo simulations or by using the response surface method [23], [41]. The latter approach is very effective and is based on the approximation of the surface of $\log(IM_{LS})$ in the hyperspace of the significant random variables, by a hyper-plane, whose coefficients are determined by a least square regression on a set of numerical experiments. If $N$ is the number of random variables, $M=2^N$ models are defined by a complete factorial combination at two levels, in which each variable assumes values corresponding to the confidence levels of 16% or 84%. The matrix $Z$ ($M$ rows $\times$ $N$ columns) collects in each row the combination of values assumed by each standard normalized random variable (-1 for confidence level 16%, +1 for confidence level 84%).

For the $i$-th model, the capacity curve $\lambda_i(D)$ is obtained and the Limit States are fixed ($D_{LS,i}$, $k=1,..,4$). By considering a generic LS, the value $IM_{LS,i}$ is evaluated by the median seismic demand $S_\alpha(S_\delta)$, using (3). The vector $Y$ ($M$ rows) collects the values $log(IM_{LS,i})$, $i=1,..,M$. The angular coefficients of the hyper-plane are obtained as:

$$\alpha = (Z^TZ)^{-1}Z^TY$$  \hspace{1cm} (13)

The dispersion $\beta_C$ is given by:

$$\beta_C = \sqrt{\alpha^T\alpha}$$  \hspace{1cm} (14)

$\beta_T$ – Uncertainty on the Limit State thresholds

The definition of the LS thresholds is also subjected to dispersion, because models adopted for the evaluation of the capacity curve are simplified and the displacements $D_{LS}$ usually derives from a heuristic approach.

Considering the median capacity curve, obtained by using the mean values of the $N$ random variables, $D_{LS}$ ($k=1,..,4$), usually distributed as in Figure 3a, can be assumed as median values. Proper distributions should be defined for these random variables, which take into account that in a single building the $k$-th DS could be reached a little bit before or after the median value $D_{LS}$. It is reasonable to assume the four distributions do not intersect. LS1 is always in the “elastic” branch of the capacity curve. LS2 is in the first part of the “plastic” branch and it is not reasonable to assume it moves too much further the median value $D_{LS}$. The position of LS3 is very variable and sometimes it occurs even for a low value of the displacement, but the possible intervals of LS2 and LS3 can be separated.

The use of a Beta distribution seems to be the best option, but for the sake of simplicity, due to the large number of uncertainties involved in a seismic risk analysis, very simple uniform distributions are suggested, which probably lead to a slight overestimation of $\beta_T$. Figure 7 shows a proposal, with the indication, for a generic LS, of the 16% and 84% confidence levels of $D_{LS}$, named $D_{LS,16}$ and $D_{LS,84}$; they are simply obtained by moving from the median value, on the left and right side, of 2/3 of the semi-wide of the uniform distribution.

For each LS, it is then necessary to evaluate $IM_{T,16}$ and $IM_{T,84}$ that corresponds to a displacement demand equal to $D_{LS,16}$ and $D_{LS,84}$ on the median capacity curve of the considered class of buildings, by using the median response spectrum $S_\alpha(S_\delta)$. The dispersion $\beta_T$ is given by:

$$\beta_T = 0.5 [\log(IM_{T,84}) - \log(IM_{T,16})]$$  \hspace{1cm} (15)
IV. DERIVATION OF FRAGILITY FUNCTIONS

After a proper building classification, tailored to data already available or that can be acquired through the survey, fragility functions can be defined by using existing ones or developing new customized curves.

In the first case, for each building class, the available functions have to be collected and examined. After the assignment of a subjective probability to each one (logic tree approach), related to its reliability and coherence with the considered building class, the fragility function can be obtained by a simple weighted summation. The use of existing fragility functions must consider several critical aspects. When a fragility function refers to a broader class of buildings, the dispersion $\beta_{LS}$ should be reduced and it is worth considering if the mean value $\bar{M}_{LS}$ has to be modified (if the behavior of the subclass is better or worse than the average). When a fragility function refers to a subclass of buildings, within the class of interest, it would be necessary to have fragility functions for the other subclasses (in the other branches of the logic tree), otherwise the obtained final fragility function would result biased; as an alternative, the dispersion $\beta_{LS}$ should be increased and the mean value $\bar{M}_{LS}$ properly modified, on the base of expert judgment.

Next sections give some hints for the development of new fragility functions. This can be done either by empirical data, if a robust database of damage observations is available in the area or in other regions where built environment has similar characteristics, or by analytical data, by the definition of mechanical models representative of each building class and able to assess the dispersion due to the variability of seismic behavior in the class.

A. From empirical/macroseismic data

In this ambit, Lagomarsino and Giovinazzi [14] have proposed a macroseismic vulnerability model, which can be considered an expert elicitation method. It is directly derived from the European Macroseismic Scale [37], which defines six vulnerability classes (named from A to F) and various building types (seven of them related to masonry buildings).

It is worth noting that macroseismic scales are not instrumental based and they implicitly contain a vulnerability model. If a building class is considered, the linguistic definitions of EMS98 may be translated in quantitative terms, by the fuzzy set theory, and an incomplete Damage states Probability Matrix (DPM) is obtained. The completion is made by using the binomial probability distribution. The vulnerability is synthetically expressed by a vulnerability curve [42], which gives the mean damage $\mu_D$ as a function of the macroseismic intensity $I$:

$$\mu_D = 2.5 + 3 \tanh \left( \frac{I+6.25V-12.7}{q} \right) \quad (0 \leq \mu_D \leq 5)$$

where: the vulnerability index $V$ and the ductility index $Q$ are parameters representative of the seismic behavior of a group of homogeneous buildings.

The vulnerability index has been defined to vary between 0 and 1 for the six vulnerability classes of EMS98. To each building class a plausible range of values of $V$ is associated, defined by a proper membership function, according to the fuzzy set theory [43]; Table 2 shows intervals for each class of maximum plausibility. The ductility index is equal to 3, in order to obtain the best fit.

Table 2. Ranges of maximum plausibility for vulnerability index of the six EMS98 classes

<table>
<thead>
<tr>
<th>Class</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>0.84-0.92</td>
<td>0.68-0.76</td>
<td>0.52-0.60</td>
<td>0.36-0.44</td>
<td>0.20-0.28</td>
<td>0.04-0.12</td>
</tr>
</tbody>
</table>

Fragility functions, in terms of macroseismic intensity $I$, can be evaluated by the binomial distribution:

$$p_{LS} = \sum_{i=k}^{s} p_{LS} \quad (k = 1...5)$$

$$p_{DSS} = \frac{5!}{k!(5-k)!} \left( \frac{\mu_D}{5} \right)^k \left( 1 - \frac{\mu_D}{5} \right)^{5-k} \quad (k = 0, 5)$$

Limit States can be identified on the vulnerability curve as points for which $p_{LS}=0.5 (k=1...5)$. The vulnerability curve is, for the macroseismic method, analogous of the capacity curve for the analytical ones. Figure 8a shows vulnerability curves, with LS thresholds, for the central (white expected) values for the six EMS98 vulnerability classes. Figure 8b shows the correspondent fragility functions of DSS for Class B ($V=0.72$).

If a proper correlation law between intensity and PGA is assumed, the fragility functions in terms of PGA are obtained; it is worth noting that, given the high number of uncertainties involved in the process, all macroseismic intensity scales may be assumed equivalent to this end. Many correlations may be found in literature, which have been calibrated in different areas and are usually in the form:

$$I = a_1 + a_2 \log(PGA)$$

Figure 8c shows the fragility functions in terms of PGA, having assumed two different correlation laws: a) Faccioli and Cauzzi [44], described by (16) with $a_1=6.54, a_2=4.51$ (PGA in m/s²); b) Murphy and O’Brien [45] for Europe ($a_1=7, a_2=4$), which gives higher values of PGA for I>8, if compared to the former one. It is worth noting that fragility functions looks very similar to a lognormal cumulative distribution; the dashed lines represent the best fit, which is obtained for all LSs by the following values of the dispersion: a) $\beta_{LS}=0.54$; b)
It is worth noting that fragility functions in Figure 8 refer to the central value of $V$ for Class B and can be considered representative of a subset of buildings in the class. In order to consider the whole class, the range of plausible values of the vulnerability index $V$ must be considered. As an example, Figure 9a shows fragility functions of LS4 in terms of macroseismic intensity, for the extreme plausible values of the vulnerability index $V$ and for the two values that define the interval of maximum plausibility (Table 2). Figure 9b shows the fragility functions in terms of PGA (considering Murphy and O’Brien correlation law). The fragility function of the whole vulnerability class is obtained by a convolution of all plausible fragility functions; the result is the dashed line in Figure 9b, which is well fitted by a lognormal cumulative distribution with dispersion $\beta_{LS} = 0.64$ (0.57 in case of Faccioli and Cauzzi correlation law). The dispersion is increased a little because in this case the fragility function represents the behavior of all different building of the class, instead of a small sub-set of these.

EMS98 proposes a classification of buildings into various types, according to masonry and horizontal diaphragms characteristics. The seismic behavior of these macro-typologies can include two or even more vulnerability classes, each one with a different subjective probability (see Fig. 5 in [14]). The corresponding vulnerability functions for each DS can be obtained with the procedure described above and showed in Figure 9b. As an example, if the building class involves two EMS98 vulnerability classes, the value of $\beta_{LS}$ for the two considered correlation laws, increases to: a) 0.62; b) 0.7.

Once the building classes have been defined by a proper list of tags from the taxonomy, fragility functions may be derived through the macroseismic method by defining a proper membership function for the vulnerability index (range of plausible values). The range can be very wide, if the building class is generic, while can be very narrow, smaller than that of a single EMS98 vulnerability class, if much information is available.

The general format of the macroseismic vulnerability method can also be used when empirical data are available. In this case, for a specific building class (defined by data acquired and by the constructive characteristic of the built environment in the area where damage survey was made), the DSs distribution (and thus the mean damage $\mu_D$) is supposed to be known for one or more values of the macroseismic intensity. If only one point of the vulnerability curve is available, the vulnerability index $V$ can be fitted and, for each LS, the corresponding $IM_{LS}$ can be evaluated and a proper value of $\beta_{LS}$ can be assumed, by considering the variability of behavior in the class. If damage data are available for more
values of the intensity, both $V$ and $Q$ can be fitted. After the conversion into fragility functions, the values of $\beta_{LS}$ may be directly fitted.

B. From analytical methods

The use of simplified mechanical models presents the following main advantages: a) fully employ all results of PSHA (instrumental IMs, seismic input in the spectral form); b) keep explicitly into account the various parameters that determine the structural response (and evaluate accurately the uncertainty propagation). However, the reliability of the vulnerability assessment is affected by the capability of the model to simulate the actual seismic response of the examined class; to this end simplified models must be validated and calibrated with observed damage or results from more sophisticated models.

As far as global response of existing masonry buildings is concerned, in-plane behavior of masonry walls can be modeled by an Equivalent Frame (EF), made by vertical piers (the columns) and horizontal spandrels (the beams), connected by rigid nodes of non-zero size. The generalized actions in masonry elements, all along the pushover analysis, depend on the relative stiffness and strength of piers and spandrels. The solution can be obtained only numerically, while analytical simplified models can make reference to two limit conditions: 1) Strong Spandrels Weak Piers (SSWP), which corresponds to the shear-type frame model and is associated to the occurrence of a soft-story failure; 2) Weak Spandrels Strong Piers (WSSP), in which full height piers (from the base to the top) work like fixed-end cantilevers and fail at the base due to axial and bending failure (rocking with crushing at the toe).

Figure 10 summarizes the effects of the coupling effectiveness of masonry piers, both in terms of deformed shape at collapse and distribution of the generalized forces (shear $V$ and bending moment $M$), in a masonry building subjected to seismic loads, passing from the case of very weak spandrels (WSSP) to the shear-type idealization (SSWP). The effects on the capacity curve, in terms of overall strength, stiffness and displacement capacities, are also shown.

![Figure 10](image)

Fig. 10. Influence of stiffness and strength of piers and spandrels on the capacity curve: generalized internal forces ($V$, $M$) in masonry piers and damage patterns according to the EF model and the two limit conditions of SSWP and WSSP (adapted from Tomaževič [46])

It is evident, in particular for medium and high-rise buildings, the range of variation of the seismic response is significant; thus, it is necessary to be able to catch these different behaviors for a reliable assessment. Usually, the presence of specific constructive details plays a crucial role in addressing the choice of the correct intermediate behavior between the two limit idealizations (WSSP and SSWP). In general, the SSWP hypothesis is consistent with new masonry buildings, in which masonry spandrels are always connected to lintels, tie beams and floor slabs, made of steel or reinforced concrete. On the contrary, in ancient buildings spandrels are in many cases quite weak elements, as lintels are in timber or made by masonry arches, tie beams are very rare and horizontal diaphragms are flexible (e.g. due to the presence of vaults or wooden floors). In these cases the behavior is closer to WSSP.

Among the different mechanical models proposed in literature (as mentioned in §II), in this section the DBV-masonry (Displacement Based Vulnerability) method is suggested. It was originally proposed in [47] with some further modifications [23], [24]. The analytical formulation makes reference to the SSWP model, under the simplified hypothesis, in the evaluation of the total base shear, that all masonry piers fail at the same time, which is true if they are more or less of the same size and the building is regular in plan. The vulnerability of actual buildings, which do not meet these hypotheses, is estimated applying proper corrective factors. Similarly it is possible to evaluate the capacity curve of buildings characterized by EF or WSSP behavior.

An alternative to the use of simplified analytical models is to assume, for each building class, one or more then one completely defined prototype buildings, and to perform static pushover or incremental dynamic analyses with detailed MDOF nonlinear models. The variability of seismic response can be evaluated by analyzing the uncertainties propagation of model parameters and/or by considering a proper number of different prototypes, representative of the class.

This approach can be more detailed because specific constructive details of buildings in the region under investigation may be taken into account explicitly. However, it is strongly dependent from the choice of prototypes and it is necessary to be sure they are really representative of all the building stock.

C. From hybrid approaches

Hybrid approaches are based on a combination of the methods described in §IV.A and §IV.B.

In particular, empirical data in terms of macroseismic intensity can be interpreted by means of the macroseismic vulnerability method [14], [42], by fitting, for each defined building class, the two free parameters: the vulnerability index $V$ and the ductility index $Q$. Then, fragility functions can be obtained through a proper I-PGA correlation. Simplified or detailed mechanical models provide fragility functions by a complementary approach.

The comparison provides a cross-validation of the two methods and helps in the definition of more reliable function
for the seismic risk analysis. Depending on the specific case, a different degree of reliability can be ascribed to the two approaches.

V. CONCLUSION

The paper presents two procedures to derive fragility curves for masonry buildings. For the first, it is important to note that each component of the dispersion coefficient can be evaluated analytically, differently from other studies that numerically estimate just one component (due to the demand) of the overall uncertainty. In the paper also the definition of a suitable taxonomy and the criteria to be adopted for a consequent classification of the built environment are discussed. Moreover, it is worth noting that the combined use of the two approaches may give interesting results:

- the analytical model, that has the advantage to keep explicitly into account the various parameters that determine the structural response of each vulnerability class, allows a more detailed classification of the assets present in hazard zones that are thereby subjected to potential losses;
- the expert elicitation based models are obtained from observed damage and directly correlated to the actual seismic behavior of buildings.

Therefore, the latter becomes the fundamental support to make reliable the results from the analytical method.

ACKNOWLEDGMENT

This work was funded by the project ReLUIS 2014-2018 (www.reluis.it), supported by Italian Civil Protection Agency.

REFERENCES


