Multiple Choice Question Tests – Advantages and Disadvantages

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***Abstract*—** In this paper we shall describe situations in which the multiple choice question tests are better than the standard tests. On the other hand, there are situations in which classical tests are more convenient. As an example we shall use tests in mathematics at the Faculty of Finance and Accounting at University of Economics in Prague. We shall consider the tests in mathematics in entrance examinations and the tests in the basic course of mathematics for the first year. For this analysis we shall use some probability methods.

***Keywords*—** Multiple choice question tests, tests in mathematics, probability methods.

I. Introduction

MULTIPLE QUESTION TESTS are widely used in testing knowledge of students. One of the advantages of such type of test is that the results can be evaluated quite easily even for large number of students. On the other hand, a student can obtain certain number of points in the test purely by guessing the right answers and this fact affects reliability of the test and should be considered in interpretation of test scores. This problem is addressed in education research – see Premadasa (1993), Klufa (2012).

An analysis of a multiple choice question test from probability point of view is provided in this paper. This test is for example used for entrance examinations at University of Economics – see Klufa (2011). Note that standard (no multiple choice questions) tests are used for checking knowledge of students in mathematics courses at University of Economics – for analysis of such test see (Otavova and Sykorova, 2014), but regarding entrance examination, multiple choice questions are preferred so that the results of tests can be obtained quickly and there is clearly no impact of any subjective factor in evaluation. Similar problems are in Moravec, Štěpánek and Valenta (2014), Brozova and Rydval (2013).

Entrance examinations at the Faculty of Finance and Accounting at University of Economics in Prague include mathematics and English. Test in mathematics has 10 questions for 5 points and 5 questions for 10 points (100 points total). Each question has 5 answers. Test in English has 50 questions for 2 points (100 points total). Each question has 4 answers. Questions are independent (one answer is correct), wrong answer is not penalized. We provide an answer to the following questions (under assumption of random choice of answers): what is probability that number of right answers exceeds given number, what is expected number of right answers, what is standard deviation, and finally what is a risk of success of students with lower performance levels.

II. Methods

Multiple choice question tests (the test has n questions, each question has m answers) are applied for the entrance examinations at the Faculty of Finance and Accounting at University of Economics in Prague. Therefore a model of binomial distribution can be used for the entrance examinations. From probability point of view a multiple choice question test means:

Let us consider n independent random trials having two possible outcomes, say “success” (right answer) and “failure” (wrong answer) with probabilities p and $(1-p)$ respectively. Probability of correctly answered question p (under assumption that each of m answers in particular question has the same probability and one answer just is correct) is $p=1/m$.

Let us denote X as number of successes (right answers) that occur in n independent random trials. X is random variable distributed according to the binomial law with parameters n and p. Probability that number of successes is k ($k=0, 1, 2, \ldots, n$) is (see e.g. Marek (2012))

$$P(X=k)=\binom{n}{k}\,p^k\,(1-p)^{n-k} \tag{1}$$

The expected value and the standard deviation of random variable X distributed according the binomial law with parameters n and p is

$$E(X)=np, \quad \sigma(X)=\sqrt{D(X)}=\sqrt{np(1-p)} \tag{2}$$

where $D(X)$ is dispersion of random variable X.

The distribution function of random variable X distributed according to the binomial law with parameters n and p is

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$$F(x) = 0, x < 0, \qquad F(x) = \sum_{k=0}^{[x]} \binom{n}{k} p^k (1-p)^{n-k}, x \geq 0 \qquad (3)$$

where *[x]* is integer part of *x*.

III. Entrance Examinations in Mathematics

Entrance examinations in mathematics have 10 questions for 5 points and 5 questions for 10 points (100 points total). Questions are independent. Each question has 5 answers; the wrong answer is not penalized. Under assumption that each answer has a same probability, probability of right answer is *p=1/5.*

Example 1. Under assumption of random choice of answers we shall find probability that number of points in the test in mathematics is 15.

Let us denote

Y = number of points in the test in mathematics
X_1 = number of right answers in the first 10 issues
X_2 = number of right answers in 10-points tasks

It holds

$$P(Y=15) = P[\,(X_1=1 \cap X_2=1)\ U\ (X_1=3 \cap X_2=0)\,] =$$
$$= P[\,(X_1=1 \cap X_2=1)\,] + P[\,(X_1=3 \cap X_2=0)\,]$$

Random variables X_1, X_2 are independent, therefore we have - see e.g. Rényi (1972)

$$P(Y=15) = P(X_1=1)\,P(X_2=1) + P(X_1=3)\,P(X_2=0)$$

Random variable X_1 has binomial distribution with parameters $n=10$ and $p=0{,}2$. Random variable X_2 has binomial distribution with parameters $n=5$ and $p=0{,}2$. According to (1) we obtain

$$P(Y=15) = \binom{10}{1} 0{,}2\ 0{,}8^9 \binom{5}{1} 0{,}2\ 0{,}8^4 + \binom{10}{3} 0{,}2^3\ 0{,}8^7 \binom{5}{0}\ 0{,}8^5 = 0{,}175922$$

Analogously, we can calculate the probability $P(Y=k)$ for other $k=0, 5, 10, 15, \ldots, 95, 100$ (see Table 1 and Figure 1). For this calculation we used software Mathematica (Statistics 'Discrete Distributions') – see Wolfram (1996).

Points in the test	Probability	Points in the test	Probability
0	0,035184	55	0,002890
5	0,087961	60	0,000957
10	0,142937	65	0,000275
15	0,175922	70	0,000067
20	0,174547	75	0,000014
25	0,146098	80	0,000002
30	0,105227	85	3×10^{-7}
35	0,066057	90	2×10^{-8}
40	0,036467	95	1×10^{-9}
45	0,017761	100	3×10^{-11}
50	0,007634	Sum	1,000000

Tab. 1 Distribution of number of points in the test in mathematics

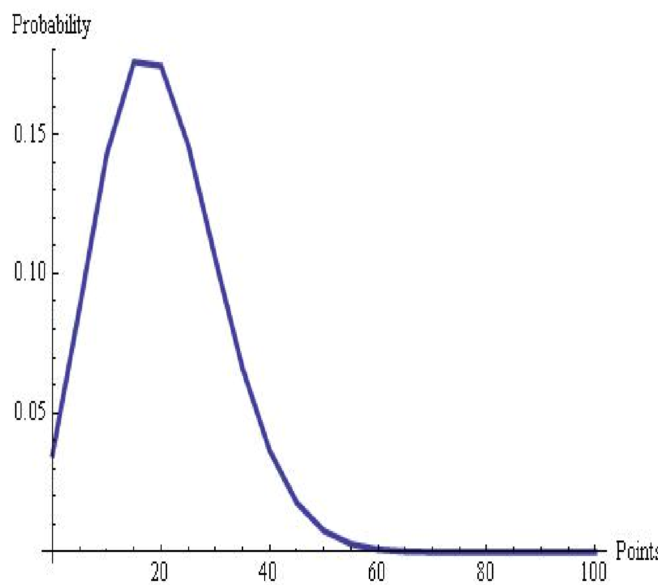

Fig. 1 Distribution of number of points in the test in mathematics (polygon)

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Example 2. Under assumption of random choice of answers we shall find probability that number of points in the test in mathematics is

(a) 30 and more,
(b) 40 and more,
(c) 50 and more.

(a) Using notation from example 1 we have - see e.g. Rao (1973)

$$P(Y \geq 30) = 1 - P(Y<30) = 1 -$$

$$P[(Y=0) \cup (Y=5) \cup (Y=10) \cup (Y=15) \cup (Y=20) \cup (Y=25)] =$$

$$= 1 - [\, P(Y=0)+ P(Y=5)+ P(Y=10)+ P(Y=15)+ P(Y=20)+ P(Y=25)\,]$$

Finally from Tab.1 we obtain

$$P(Y \geq 30) = 1 - 0{,}762649 = 0{,}237351.$$

Under assumption of random choice of answers almost a quarter of students get the test score 30 or more points.

(b) Analogously, we obtain

$$P(Y \geq 40) = 1 - P(Y<40) =$$

$$= 1 - [\, P(Y=0)+ P(Y=5)+ P(Y=10)+ P(Y=15)+ P(Y=20)+ P(Y=25) + P(Y=30) + P(Y=35)\,]$$

Finally from Tab.1

$$P(Y \geq 40) = 1 - 0{,}933933 = 0{,}066067.$$

Under assumption of random choice of answers approximately 6,6% of students get the test score 40 or more points.

(c) Finally

$$P(Y \geq 50) = 1 - 0{,}988161 = 0{,}011839.$$

Under assumption of random choice of answers approximately 1,2% of students get the test score 50 or more points.

Example 3. Under assumption of random choice of answers we shall find expected number of points in the test in mathematics and mode.

Using notation from example 1 we have

$$Y = 5\, X_1 + 10\, X_2$$

Therefore - see e.g. Anděl (1978)

$$E(Y) = E(5X_1 + 10X_2) = 5\, E(X_1) + 10\, E(X_2)$$

According to (2) we obtain (e.g. $E(X_1) = 10 \,.\, 0{,}2 = 2$)

$$E(Y) = 5 \,.\, 2 + 10 \,.\, 1 = 20.$$

Expected number of points in the test is 20. The mode is the most probable number of points. From Tab.1 is

$$\hat{y} = 15.$$

I. IV. Conclusion

Entrance examinations at the Faculty of Finance and Accounting at University of Economics in Prague include mathematics and English. Probability that number of points from test in mathematics is 50 and more is 0,011839 (see example 2). Analogously, we can calculate this probability for test in English. We obtain 0,000122. That means (both tests are independent: 0,011839 x 0,000122 = 0,000001) that approximately one student from million (under assumption of random choice of answers and using 50 points as a cut-off value for successful completion in each test) successfully makes the entrance examinations at the Faculty of Finance and Accounting at University of Economics by pure guessing the answers.

Multiple choice question tests are optimal for entrance examinations at University of Economics. These tests are objective (there is clearly no impact of any subjective factor in evaluation). Moreover, results can be evaluated quite easily for large number of students. From results of this paper follows that risk of acceptance students with lower performance levels is negligible.

On the other hand, number of students in the basic course of mathematics is not large. In this case is better use the standard tests. These tests (the solution of concrete examples) examine the students' knowledge of mathematics better than the multiple choice question tests.

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