Ground motions due to seismic forces under Hilbert-Huang transformation

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Abstract—The recent catastrophic experiences have shown that seismic forces and the ground responses are still a challenging concern and more efforts are needed to improve the engineering art and the praxis. This study explores the use of the Hilbert-Huang Transform HHT for analyzing earthquake recordings and the associated responses of soft-soils deposits in Mexico City where, in a single earthquake, the motions at one site can easily be 10 times stronger than at a neighboring position, even when their distance from the ruptured fault is the same. Based on the numerical findings, geotechnical-geosphere engineers, insurance companies, and emergency-management officials could use this tool to elucidate about complex/anomalous shaking-amplification patterns. The use of emerging techniques will not only help to save lives and protect property in future Mexican quakes but will also help to improve understanding of seismic hazards in many earthquake-prone regions of the world.

Keywords—Earthquake response, Hilbert-Huang Transform, Mexico City clays, Response spectra analysis, Site effects, Soft soils, Time series analysis.

I. INTRODUCTION

Soil effects are a very important contributor to human suffering and damage during earthquakes. There are mainly two types of problematical ground response when the ground starts to shake: i) the soil fails, cracks and moves laterally/horizontally and, ii) the soil changes the character of the ground shaking by amplifying it and making it more destructive.

In a single earthquake the shaking at one site can easily be 10 times stronger than at another site, even when their distance from the ruptured fault is the same. Scientists have assumed that local geologic conditions (the softness of the rock/soil near the surface and the thickness of the sediments above hard bedrock) are the cause of this difference in shaking intensity, but they have not been certain of the particular conditions that are most responsible, and the degree to which they affect the different consequences levels.

The consideration of surface soil amplification is based on motions recordings and their interpretation drives theories and conclusions about the phenomena. This research explores the spectra-based methods for studying rock/soil accelerations and illustrates how this tactic could lead to a distorted picture of nonlinear site amplification/response [8], [21].

As an alternative to the traditional spectrum-based approach, in this study the Hilbert-Huang Transform HHT [9].

A method for nonstationary data processing, is used to describe soil response from earthquake recordings. The HHT is an empirically-based data-analysis method. Its basis of expansion is adaptive, so that it can produce physically meaningful representations of data from nonlinear and non-stationary processes. The proposed scheme of HHT analysis for discerning soil nonlinearity permits to infer aspects related to duration, frequency and intensity of shakings where significant nonlinear behaviors can develop. Different intensity motions recorded in clayey soft deposits in Mexico City are examined to demonstrate the validity and effectiveness of the HHT system in estimating site effects.

II. COMPLEX SOILS, COMPLEX SHAKING

Useful study of the site effects requires comprehensive computer simulations of potential earthquakes on a case-by-case basis and deep analyses of soils-conditions—monitored-responses to validate simulations and models. In general, evaluating the amplification levels requires i) determining the response spectrum on the bedrock, (generally, materials with shear wave velocity $V_s$ greater than about 500m/s) ii) characterizing/organizing the response spectrum on the soils deposits/sediments and iii) a framework examination for developing numerical relations that model the ground motions. Then, under a simplistic position, an earthquake ground motion on the ground surface can be estimated efficiently by multiplying the rigid-base (bedrock) spectra by a response spectrum ratio that describes amplification factors (frequency dependent) including specific characteristics of the soil layer(s).

If an attenuation formula (for estimating the acceleration response spectrum on the bedrock from the earthquake source—epicenter) and the transfer function are applied, in theory, it is possible to compute the response on the ground surface for any seismic input. Following this analysis scheme, the relations between the acceleration response spectrum of the bedrock and the response spectrum ratio of the ground surface are completely described. The relation between the acceleration response spectra of the bedrock and those of the ground surface is

$$S_{Ab}(T, h, G_c, E_c, A_c) = S_{Ab}(T, h) \cdot R(T, h, G_c, E_c, A_c)$$

where $S_{Ab}(T, h, G_c, E_c, A_c)$ is a linear/nonlinear acceleration response spectrum of the ground surface, $S_{Ab}(T, h)$ is the acceleration response spectrum of the bedrock, $R(T, h, G_c, E_c, A_c)$ is the acceleration response spectrum ratio with a linear/nonlinear response characteristics of surface soil,
$T$ is the period, $h$ is the damping ratio, $G_e$ is topographical classifications, $E_e$ is the earthquake type, and $A_e$ is the earthquake ground motion intensity described by peak acceleration. By using topographical classification, earthquake type and earthquake ground motion intensity, a simple estimate for the acceleration response spectrum ratio of surface soil with a nonlinear characteristics is

$$R(T, h, G_e, E_e, A_e) = \tilde{H}_L(T) \cdot \alpha(T, h, G_e, E_e, A_e)$$

where $\tilde{H}_L(T)$ is the smoothed transfer function that represents a weighted average of the (usually) linear response transfer functions, and $\alpha(T, h, G_e, E_e, A_e)$ are coefficients deduced from topographical classifications, earthquake types, and earthquake ground motion intensities. The described spectrum ratio procedure seems to be able for reproducing acceleration response spectrum at any point contained in the soils system.

Through the analysis on particular cases, where the predicted and experienced responses differ dangerously, it has been concluded the simplistic method is not competent to reproduce acceleration response spectrum of complex soils deposits and multifaceted shaking generation. In this investigation it is argued that the postulate about amplification ratios is very useful but the interpretation environment (frequency/period dominion) is possibly distorting the implicit information in accelerations time series and originating mistaken numerical/linguistic conclusions.

### III. THE HILBERT-HUANG TRANSFORM

The Hilbert-Huang Transform HHT was proposed by [9] and consists of two parts: i) Empirical Mode Decomposition EMD, and ii) Hilbert Spectral Analysis HS or Hilbert transformation. With EMD any data set can be decomposed into a finite number of intrinsic mode functions IMFs. An IMF is described as a function satisfying the following conditions: i) the number of extrema and the number of zero-crossings must either equal or differ at most by one; and ii) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. An IMF admits well-behaved Hilbert transforms [7].

For an arbitrary function $x(t)$ in linear programming-class [19], its Hilbert transform $y(t)$, is defined as,

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x(t’)}{t-t'} dt'$$

where $P$ indicates the Cauchy principal value. Consequently an analytical signal $Z(t)$, can be produced by,

$$Z(t) = X(t) + jY(t) = a(t)e^{j\theta(t)}$$

Where

$$a(t) = \left[X^2(t) + Y^2(t)\right]^{\frac{1}{2}}$$

$$\theta(t) = \arctan \frac{Y(t)}{X(t)}$$

Are the instantaneous amplitude and phase of $X(t)$, respectively. Since Hilbert transform $Y(t)$ is defined as the convolution of $X(t)$ and $1/t$ by Eq. 3, it emphasizes the local properties of $X(t)$ even though the transform is global. In Eq. 4, the polar coordinate expression further clarifies the local nature of this representation. Using Eq. 4, the instantaneous frequency of $X(t)$ is defined as,

$$\omega(t) = \frac{d\theta(t)}{dt}$$

However, there is still considerable controversy on this definition. A detailed discussion and justification can be found in [9]. EMD is a necessary pre-processing of the data before the Hilbert transform is applied. It reduces the data into a collection of IMFs and each IMF, which represents a simple oscillatory mode, is a counterpart to a simple harmonic function, but is much more general. We will not describe EMD algorithm here due to the limitation of the length of the paper. The readers are referred to [9] for details.

By EMD, any signal $X(t)$ can be decomposed into finite IMFs $imf_i(t)$ $(i = 1, \ldots, n)$, and a residue, $r(t)$ where $n$ is nonnegative integer depending on $X(t)$, i.e.,

$$X(t) = \sum_{i=1}^{n} imf_i(t) + r(t)$$

For each $imf_i$, let $X_i(t) = imf_i(t)$, its corresponding instantaneous amplitude, $a_i(t)$, and instantaneous frequency, $w_i(t)$, can be computed with Eqs. 5 and 6. By Eqs. 4 and 6, $imf_i$ can be expressed as the real part RP in the following form,

$$imf_i(t) = RP\{a_i(t)\exp \{i \int \omega_i(t) dt\}\}$$

Therefore, by Eqs. 7 and 8, $X(t)$ can be expressed as the IMF expansion as follows,

$$X(t) = RP \sum a_i(t) \exp \{i \omega_i(t) dt\} + r(t)$$

which generalize the following Fourier expansion

$$X(t) = \sum_{j=1}^{n} a_j e^{i\omega_j t}$$

by admitting variable amplitudes and frequencies. Consequently, its main advantage over Fourier expansion is that it accommodates nonlinear and non-stationary data perfectly. Eq. 9 enables us to represent the amplitude and the instantaneous frequency as functions of time in a three-dimensional plot, in which the amplitude is contoured on the time-frequency plane. The time-frequency distribution of amplitude is designated as the HS or simply Hilbert spectrum, denoted by $H(\omega, t)$. For a comprehensive illustration of the HHT-seismic approach, Reference [6] is suggested.

### IV. HHT ON ACCELEROMETERS

As an example of the Huang transformation and the Hilbert
spectra applied to earthquake recordings, the accelerogram of the September 14th 1995 earthquake recorded in CDAO site, soft-soils deposit in México City, is studied. From this signal, see its HS and its corresponding Fourier spectrum in Figure 2. Fourier spectrum indicates that the frequency content of the waves is spread out with the maximum spectral amplitudes at 0.3 and 0.9 Hz, with a well-defined peak amplitude value of ~170 cm/s at 0.3 Hz, the dominant frequency.

On the other hand, the Hilbert spectra HS (bottom of Figure 2) shows quantitatively the temporal-frequency distribution of vibration characteristics in the ground-motion recording.

A detailed description of the motion can be identified from the 2D display: i) two sets of ground motion in the time intervals 70–75 and 135–150s consisting of low-frequency signals between 0.25 to 0.80 Hz with intensities around 28 and 14 cm/s², respectively, ii) the highest intensity Imax (~28 cm/s²) at 0.4 Hz is surrounded by an important concentration zone (iso-acceleration curves ~ 20 cm/s²) on a wider frequency band (from 0.2 to 0.5 Hz) and iii) the second motion arrival, with accelerations on the middle of the intensity-scale registered, covers frequencies around 1.0 Hz.

Since these amplitudes and frequencies are quite often used as an index to characterize and define the seismic inputs, the distinction between values based on the two different methods can be critical to the seismic design, retrofit guidelines and codes. It is worth further analyses to see which value (Fourier or HHT-based) is more appropriate for structural design, but the differences between frequency-energy content can be critical when site effects are being analyzed. The full understanding of these characteristics is a subject of continuing study, for now, the potential exists for a useful quantitative measure of a motion’s input energy to structural and geotechnical systems.

A. Interpreting site effects

The numerical procedures, all of them based on the wave equation, have been proposed to take into account of several of the various aspects of amplification due to soils characteristics. However, the experimental methods are the most recommended for singular materials and/or erratic stratigraphies because the records of earthquakes enclose specific information to model more rationally site effects and the response of structures. The experimental techniques use the spectral ratios of motions records (weak and strong) to obtain from an accelerogram.
conclude about behaviors (earthquake ratios, peaks ground accelerations ratios, resonance frequency). Amplification of ground motion due to geological and soil conditions could be extrapolated by means of linear methods. However, recent analyses [13], [20], have demonstrated important presence of nonlinear behaviors. There is a bulk of technical information showing that both linear and nonlinear approaches are capable of explaining this behavior and, with a certain degree of accuracy, computing the response of soil deposits under earthquake loading. This sole fact does not elucidate the rather generalized controversy about the significance of nonlinear soil effects [1], [5], [22]. In the authors’ opinion the direct way to clarify this question is to look into hard seismological evidence showing the influence of such phenomenon. Thus, it boils down to decode the encrypted information that records of ground motions possess.

For the Mexican heterogeneous soils deposits where analytical solutions for determining site effects do not seem valid, it is desirable to use the in situ information. Mexico City has already suffered a destructive earthquake (Michoacán event Sept 19th 1985) thus detailed macroseismic observations are available. More than 50 years of monitoring makes possible estimate the site-specific transfer functions experimentally through minor, intermediate and strong earthquake records [2], [10], [16].

Being the HHT ideal for studying nonlinear /nonstationary processes, in the following the recordings from two Lake Zone sites of Mexico City are transformed into their HS to illustrate i) the possible misinterpretations originated by traditional spectra approach and ii) the advantages of examining accelerograms in the I-F-T (intensity-frequency-time) space (Hilbert spectra HS).

CDAO and SCT stations are the soils deposits studied (Figure 3). For the UBC (Uniform Building Code) and EC8 (Eurocode 8) standards, based on the Vs30 values, [3], [17], [14] SCT and CDAO are classified into the same soil class SF (UBC) / S1 (EC8): “deposits consisting- or containing a layer <100 at least 10 m thick- of soft clays/silts with high plasticity index (> 40) and high water content”.

In comparison with data analysis in a transformed domain, major characteristics of the data can be simply and directly obtained from the analyses in the time domain. A typical example to illustrate this point is the Michoacán, Mexico earthquake of 19 September 1985. While many factors contributed to the enormous damage in the city, such as design and period of buildings, year of construction, site characteristics, and population density, it nevertheless indicates that a comprehensive description of ground motion for engineering use likely needs more information, such as frequency content associated with the peak motion. The original data from the two lake-zone sites, SCT and CDAO, and a rock-like site (CU station), after decomposing by EMD (12 IMFs plus a residue) generate the corresponding HS depicted in Figure 4.

It can be assumed, based on these Hilbert spectra, that significant energy in SCT and CU is related with frequencies from 0.3 to 2 Hz while the energy resides along horizontal belts below 1 Hz for CDAO. These findings are not clearly shown in other representations as Fourier spectra (top of the Figure 5). HHT can efficiently reveal frequency-energy distribution of the seismic intensity during the more than 100s registered. Higher amplitudes (zones from red to black) can be recognized for SCT site between 0.2 and 2.0 Hz, pulses-like waves are not detected. For CDAO, the energy is released from the source permanently in time, there are a few relax intervals but it is clear that CDAO registered the important seismic arrival at t= 40s with amplitude and frequency content almost constant the next 50s. Nevertheless CU and SCT are clear examples of transient processes.

![Fig. 3 Soil properties of SCT and CDAO (very soft, high water content, very compressible clays)](image-url)
Their HS show the arrivals of the energy-containing packets with increasing frequencies, within a brief span of time. In SCT and CU a low-frequency pulse-like packet is registered around 60 s, where the maximum amplitudes and the broader frequencies are activated. It is obvious that there are different accelerations readings in the soft-soil (SCT) and rock (CU) sites but their energy-time-frequency distributions are much related.

From a seismological perspective, these observations imply two major subevents in the rupture process. From the viewpoint of structural engineering, the first set of waves will cause much larger dynamic responses of long period structures than will the second, but it is important to point out that the major amplitudes are registered in the second packet. The key parameter to characterize the responses would be the duration of the shaking packets, so a proper argument about the nature of ground motions and their dependence on soil and seismic conditions should contain the triplet I-F-T (intensity-frequency-time).

The amplitude spectra of SCT and CDAO for the 1985 event (strong motion) have been extensively used for some authors (e.g. [11], [12], [15], and [18]) For quantifying the amplification of motions in the lakebed of Mexico City. They reported that in the lakebed (SCT/CDAO) as compared to the hill zones (CU), the motions were amplified by 8 to 20 times. Spectral ratios were calculated using a simple amplification function relationship and they substantiate the dominant low frequencies of the motions in Mexico City. Via this Fourier approach SCT and CDAO could be characterized as exhibiting dominant frequencies between 0.3-0.8 Hz with amplification ratios around 10 units.

The findings obtained from weak motions exhibit several distinctive characteristics, being one of the most relevant that amplification levels are significantly larger than those from strong motions.

In Figure 6 the HS for the SCT, CDAO, and CU stations after decomposing the recordings (event 7Oct. 24th 1993, Mc6.5) are shown. It can be assumed, based on these Hilbert spectra, that significant energy in SCT is related with frequencies from 0.3 to 4 Hz, CDAO with 0.3 to 1.5 and CU with 0.2 to 0.8 Hz. The energy resides along horizontal belts below 1 Hz for CDAO and CU but for SCT important energy concentrations are around a broad range of frequencies during a short period of time (spike-like). These important differences cannot be observed in the Fourier spectrum (top of Figure 7). HHT can efficiently reveal frequency-energy distribution of the low seismic intensity during the ~100s registered. Again, higher amplitudes (zones from red to black) are recognized for SCT site but they covered from 0.2 to 4.0 Hz, while CDAO behavior is very similar to that observed during the strong event: the energy is released from the source permanently in time.

Because of the amplitude accelerations, for CU cannot be
recognized a particular behavior. Its HS show the arrival of a low-energy-containing packet with frequencies under 1 Hz within a brief span of time. In SCT a pulse-like packet is registered around 50 s, and for CDAO a similar conduct cannot be concluded. It is obvious that there are considerable differences between the two soft-soil (SCT/CDAO) energy-time-frequency distributions. It is important to point out that studying the responses for strong and weak events, the obtained amplification patterns (~spectral ratio) are not related.

Observe the energy peaks and the frequency content for “weak” and “strong” events (see Figures 4 and 6). Rock-site distribution of energy changes substantially going from a localized predominant frequency at 0.4 Hz for “weak” events to a frequency-zone (0.5 to 1.5) activated during the “strong” event.

The mixed frequency content of the recordings, containing low and high frequencies, is truthfully reflected in the HS. CDAO shows a response of ~8 cm/s² at 0.2 Hz for the “weak” shaking and of ~80 cm/s² at 0.4 Hz for “strong” input. SCT, with a broader band of frequencies, depicts an intensity of 180 cm/s² at 0.85 Hz for the strong event and 18 cm/s² at 1.0 Hz for the weak motion. The SCT abnormal peaks, for example at 55 s for the Oct 24th 1993 event and at 60 s for the Sept 19th 1985 event, (cusped waveforms and high-frequency spikes) are symptomatic of a nonlinear response at some soil sites [4].

Contrary to the Fourier results (Figure 8), HHT demonstrates that the responses to the strong movement contain lower frequency levels compared with those generated during the weaker earthquake.

Even in Mexico City, an ideal case where the most common assumptions for soil amplification are fulfilled, the most accepted theories are partially verified. The theories based on Fourier schemes can reproduce the natural soil response period but fail in reproducing the amplification intensity and duration. Using HS we can avoid the misinterpretation of soil amplification patterns.

Fig. 6 HS of two soft-soils deposits (SCT and CDAO) and a rock-like site (CU), Oct. 24th 1993, Mc6.5

Fig. 7 Fourier spectra and spectral ratios for soft-soils deposits/rock-like site Oct. 24th 1993, Mc6.5 (weak shaking)
Fourier spectra ratios of the weak motions are slightly larger than strong motion ratios over the frequency range of 0.1 to 1.0 Hz (Figure 8) but this evidence cannot be interpreted as a clear indication of nonlinear response. The ground motion at SCT is amplified at its natural frequency ($f_n = 0.5$ Hz) about 10 times for the strong motion and around 20 times for the weak motion, while CDAO responses show an amplification of approximately 23 times for strong and about 12 times for the weak event, this ambiguous indication of nonlinear clay-behavior could be driven by the analysis tool rather than the actual encrypted in the recorded motions.

The mixed frequency content of the recordings, containing low and high frequencies, is truthfully reflected in the HAS. The CDAO-HAS show a maximum intensity response (~8 cm/s²) at 0.2 Hz for weak input (CU) and at 0.4 Hz for strong input (~80 cm/s²). SCTYPE-HAS, with a broader band of frequencies, depict an intensity of 180 cm/s² at 0.85 Hz for the strong event and 18 cm/s² at 1.0 Hz for the weak motion. The CDAO abnormal peaks, for example at 55 s for the Oct 24th 1993 event and at 60 s for Sept 19th 1985 event, (cusped waveforms and high-frequency spikes) are symptomatic of a nonlinear response at some soil sites [4]. Contrary to the Fourier results, HHT demonstrates that the responses to the strong movement contain lower frequency levels compared with those generated during the weaker earthquake.

Time is a key factor when site effect is being studied. Duration is one of the main parameters characterizing ground motions and the cumulative damage endured by the structures. In this sense, the advantage of using HS is that it reveals directly the duration of the event (where significant amplitudes and frequencies are activated) and showing the nonstationary characteristics of the seismic phenomena.

Vertical accelerographic array allows detecting and studying this effect. As an example see the CDAO Fourier spectra shown in Figure 10.

Noticeable differences in amplification are detected in 60 m and 30 m responses, while the spectrum at 12 m is equivalent to the surface spectrum implying that this upper layer does not amplify the motions (this is more evident under strong than weak shakings). On the other hand, the Hilbert spectra showed in the same Figure permit to outline the differences between these superficial responses and to mark the effects of the input characteristics on the amplification ratios. Variations between the HS of the record at 60 m deep and the motion at CU (outcropping) in intensities and frequency content alert about the potential impact of the input on the amplification ratios and soil-nonlinearity conclusions that have been drown in past studies that make use of Fourier analyses.

Furthermore, HHT results reveal deamplification from 12 to 0 m and because of the modification of the frequency content this superior layer can be considered as if it were a low-pass filter. In the frequency range containing most of the radiated energy (around CDAO natural frequency), the attenuation of the strong motion by hysteretic damping be larger than for weak motions.

The above arguments indicate that the HHT allows a more precise characterization of the layers behaviour as linear or nonlinear and demonstrates that Fourier analysis is not able to account some aspects of the response characteristics related with nonstationary time histories. More analyses about nonlinearity, fundamental period and soil degradation are needed but there is no doubt that the HHT is more versatile when explaining the seismic phenomena because makes use of an adequate space: [intensity, frequency, time].

Fig. 8 Spectra ratios obtained from studying the Fourier amplitude spectrum from weak and strong events

Fig. 9 Vertical accelerographic array in CDAO: HHT interpretation (Event Nov. 9th 1995, Mc: 7.5)
This study introduces the method of HHT for earthquake data analysis and investigates its rationale for studies of earthquake engineering and seismology. The results presented here reveal the HHT capabilities for analyzing nonstationary dynamic and earthquake motion recordings. The decomposed components, namely, the IMF components, may contain observable, physical information inherent to the original data and also capture the features of the response spectrum of the original data. The Hilbert spectra show the temporal-frequency energy distribution for dynamic and earthquake motion recordings precisely and clearly. For natural systems (i.e. soil deposits) where only exist measured responses, the question as to whether linear or nonlinear characteristics (of the systems) can be identified from the data cannot be answered from the traditional duality cause-effect. For seismic processes the Fourier-based spectral analyses are not adequate because they are based on linear and stationary assumptions. In this short paper we have addressed the possibility of characterizing natural systems (soil deposits) by the time-frequency variations of system signals (accelerograms represent the systems dynamics). The objective of describing data from engineering perspectives (time-frequency-intensity domain) for finding specific and indicative behavior patterns reducing the error mechanisms is addressed applying the HHT to the seismic information. Our analysis is of a preliminary nature and many issues have to be investigated rigorously but the HHT seems to have much potential for this approach.

V. CONCLUSIONS

The Fourier-Ubased spectral analyses are not adequate the HHT seems to have much potential for this approach.

REFERENCES


