Seismic behavior of fluid flows fully coupled with rectangular tank

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Abstract—Liquid-containing tanks are used to store variety of liquids. This paper provides theoretic background for specification of impulsive and convective action of fluid in liquid storage rectangular container. Numerical model of tank seismic response - the endlessly long shipping channel was obtained by using of Finite Element Method (FEM), Arbitrary Lagrangian Eulerian (ALE), Fluid Structure Interactions (FSI) formulation in software ADINA. It was considered the horizontal ground motion of the earthquake in Loma Prieta.

Keywords—arbitrary Lagrangian-Eulerian formulation, earthquake, finite element method, Fluid-structure interaction, tank

I. INTRODUCTION

Liquid-containing rectangular tanks are used to store variety of liquids, e.g. water for drinking and fire fighting, petroleum, oil, liquefied natural gas, chemical fluids, and wastes of different forms. Therefore, this type of structures must show satisfactory performance, especially, during earthquakes.

In particular, the analysis and design of liquid storage tanks against earthquake/induced action has been the subject of numerous analytical, numerical, and experimental works.

Numerous studies have been carried out about seismic behavior of ground-level cylindrical tanks. However, the conditions are not the same for underground tanks, rectangular tanks, and elevated tanks.

The seismic analysis and design of liquid storage tanks is, due to the high complexity of the problem, in fact, really complicated task. Number of particular problems should be taken into account, for example: dynamic interaction between contained fluid and tank, sloshing motion of the contained fluid; and dynamic interaction between tank and sub-soil. Those belong to wide range of so called fluid structure interactions (FSI). The knowledge of pressures acting onto walls and the bottom of containers, pressures in solid of tanks, liquid surface sloshing process and maximal height of liquid’s wave during an earthquake plays essential role in reliable and durable design of earthquake resistance structure/facility - tanks. The analysis of a coupled multi-physics system is frequently required today to understand the behaviour of the system. In particular, the analysis of problems that involve fluid flows interacting with solids or structures is increasingly needed in diverse applications including ground-supported tanks used to store a variety of liquids.

To model the behavior of solid media the Lagrangian formulation of motion is employed, whereas, for a fluid flow analysis the Eulerian formulation is usually used since it is of interest to know the behavior of the fluid. However, when considering a fluid flow interacting with a solid medium and with free surface, the fluid domain changes as a function of time, and an arbitrary Lagrangian-Eulerian (ALE) description of the Eulerian and Lagrangian descriptions. We will concentrate in this paper on the analysis of fluid flows that can deform. The flow equations are modeled using the Navier-Stokes equations of motion, and the constitutive relations of the structure are assumed to be either linear. [3-6,13]

II. MECHANICAL MODEL

The dynamic analysis of a liquid - filled tank may be carried out using the concept of generalized single - degree - of freedom (SDOF) systems representing the impulsive and convective modes of vibration of the tank - liquid system as shown in Fig. 1. For practical applications, only the first convective modes of vibration need to be considered in the analysis, mechanical model. The impulsive mass of liquid $m_i$ is rigidly attached to tank wall at height $h_i$. Similarly convective mass $m_{wi}$ is attached to the tank wall at height $h_{wi}$ by a spring of stiffness $k_{wi}$. The mass, height and natural period of each SDOF system are obtained by the methods described in [20].

$$m_i = m 2\pi \sum_{n=0}^{\infty} \frac{I_n(v_{1}/\gamma)}{v_n^3 I_n(v_{1}/\gamma)},$$

$$h_i = H \frac{\sum_{n=0}^{\infty} (-1)^n \frac{I_n(v_{1}/\gamma)}{v_n^3 I_n(v_{1}/\gamma)} v_n (-1)^n - 1}{\sum_{n=0}^{\infty} \frac{I_n(v_{1}/\gamma)}{v_n^3 I_n(v_{1}/\gamma)}},$$

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\[
m_{n} = \frac{m}{2 \tanh(\lambda_{n} H/L)} \left( \frac{\lambda_{n} H/L}{\lambda_{n}^{2} - 1} \right),
\]
\[
h_{n} = H \left( 1 + \frac{1 - \cosh(\lambda_{n} H/L)}{(\lambda_{n} H/L) \sinh(\lambda_{n} H/L)} \right),
\]
\[
k_{n} = \omega_{n}^{2} m_{n}
\]
\[
\omega_{n}^{2} = \frac{\lambda_{n}^{2}}{L} \tanh(\lambda_{n} \gamma)
\]

where \( \nu_{n} = \frac{2n + 1}{2} \), \( \gamma = H/R \). \( I_{1}(\cdot) \) and \( I_{1}'(\cdot) \) denote the modified Bessel function of order 1 and its derivate. The derivate can be expressed in terms of the modified Bessel functions of order 0 and 1 as: \( I_{1}'(x) = \frac{dI_{1}(x)}{dx} = I_{0}(x) - I_{1}(x) \).

\( \lambda_{n} \) are soliation of Bessel function of first order, \( \lambda_{1} = 1.8412; \lambda_{2} = 5.3314; \lambda_{3} = 8.5363; \lambda_{4} = 11.71; \lambda_{5} = 14.66 \) and \( \lambda_{5+i} = \lambda_{5} + 5i \) \((i = 1, 2, ...)\).

Fig. 1 liquid-filled tank modelled by generalised single degree of freedom systems.

For a horizontal earthquake ground motion, the response of various SDOF systems may be calculated independently and then combined to give the base shear and overturning moment. The most tanks have slimness of tank \( \gamma \) whereby 0.3 < \( \gamma < 3 \). Tank’s slimness is given by relation \( \gamma = H/L \), where \( H \) is the filling height of fluid in the tank and \( 2L \) is inside width of tank.

III. FEM - FLUID-STRUCTURE INTERACTION

For the fluid-structure interaction analysis, there are possible three different finite element approaches to represent fluid motion, Eulerian, Lagrangian and mixed methods. In the Eulerian approach, Eulerian, Lagrangian and mixed methods. In the mixed approaches, both the pressure and displacement fields are included in the element formulation, [1-2, 7].

In fluid-structure interaction analyses, fluid forces are applied into the solid and the solid deformation changes the fluid domain. For most interaction problems, the computational domain is divided into the fluid domain and solid domain, where a fluid model and a solid model are defined respectively, through their material data, boundary conditions, etc. The interaction occurs along the interface of the two domains. Having the two models coupled, we can perform simulations and predictions of many physical phenomena, [14, 18].

In many fluid flow calculations, the computational domain remains unchanged in time. Such the problems involve rigid boundaries and are suitable handled in Eulerian formulation of equilibrium equations [1, 11]. In the case where the shape of the fluid domain is expected to change significantly, modified formulation called Arbitrary Lagrangian-Eulerian (ALE) formulation was adopted to simulate the physical behavior of the domain of interest properly. The ALE description is designed to follow the boundary motions rather than the fluid particles. Thus, the fluid particles flow through a moving FE-mesh. Basically there are two different algorithms available for generation of possible moving mesh:

- remeshing of fluid domain, which is computationally expensive procedure,
- rezoning of FE-mesh of fluid domain. This procedure is quite fast while precise enough if no dramatic, changes of fluid domain is expected.

A. Governing Equations

Dynamic equilibrium of fluid domain involving effect of moving mesh describes modified Navier-Stokes equations. Let us to assume temperature independent problem. Then the balance of momentum by ALE formulation is

\[
\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} - \mathbf{v}_{b}) \cdot \nabla \mathbf{v} \right] = \nabla (-p \mathbf{I} + \mathbf{\tau}) + \rho \mathbf{g},
\]

where \( \rho \) is density of fluid, \( \mathbf{v} \) velocity of fluid, \( \mathbf{v}_{b} \) velocity of moving FE-mesh, \( p \) fluid pressure, \( \mathbf{I} \) unit matrix, \( \mathbf{\tau} \) stress tensor and \( \mathbf{g} \) gravity acceleration.

Dynamic equilibrium of solid domain governs balance of momentum, e.g. in Cauchy form it is

\[
\mathbf{div} \mathbf{\tau} + \rho_{s} (\mathbf{b} - \mathbf{u}) = 0,
\]

where \( \rho_{s} \) is density of solid in initial configuration, \( \mathbf{u} \) displacement, \( \mathbf{b} \) body load, \( \mathbf{\tau} \) stress tensor.

Together with traditional boundary conditions defined for fluid domain (pressure and velocity), additional special conditions are considered:

- free surface, the interface between fluid and gas,
- FSI boundary, common boundary between solid and fluid.

B. Fluid Domain, Free Surface, Boundary conditions

Dynamic boundary condition for free surface express balance of forces between interactive forces of liquid and gas

\[
f_{l}\mathbf{n} + \sigma \mathbf{K} = -f_{g}\mathbf{n},
\]

\[
f_{l}\mathbf{t} + \sigma \mathbf{K} = -f_{g}\mathbf{t},
\]

\[
f_{l}\mathbf{s} + \sigma \mathbf{K} = -f_{g}\mathbf{s},
\]

where \( f_{l} \), resp. \( f_{g} \) are forces exerted by liquid, resp. gas, \( \mathbf{t} \) a \( \mathbf{n} \) tangent and normal to FSI surface and \( \mathbf{S} \) is surface tension.
Kinematic boundary condition states the velocity at a point of free surface moves together with point of FE-mesh. Thus \((v - v_s)\cdot n = 0\).

### C. FSI Boundary, conditions

Dynamic boundary condition defines stresses at the common FSI boundary, which is opposite and equal \(\sigma_f = \sigma_s\).

Kinematic boundary condition assumes velocities and displacements of FSI boundary are the same \(v_f = v_s\), \(u_f = u_s\).

where indexes \(f\), resp. \(s\) mean fluid, resp. solid, Fig. 2.

\[
\text{Fluid} \quad n \quad \text{Pressure } p \\
\text{Structure} \quad \text{Traction } -p
\]

\[
v_f = v_s, \quad u_f = u_s,
\]

Fig. 2 common velocity and displacement of FSI boundary

### D. Discretization by Finite Elements

Any of unknown physical variables in Finite element method is express in terms of nodal values instead of field value. That causes local discontinuity of the problem, but globally, with regards to whole FE model all governing equations are satisfied.

Unknown variables (displacement, velocity and pressure) are approximated using so called shape functions \(N\).

\[
\hat{u} = NU, \quad \hat{v} = NV, \quad \hat{p} = NP,
\]

where \(U, V, \) resp. \(P\) are nodal values of initially unknown fields, \(N\) are shape functions.

Applying one of appropriate variation principle, governing equations are transformed into integral form, in which interpolations (13) are being easily incorporated and followingly proceeded in numerical calculation.

As the governing equations are basically nonlinear and time dependent, an appropriate linearization should be used together with a discretization in time domain. Plenty of methods by linearization and time discretization were published in the past. ADINA has implemented some of most popular of them [10,11,16-19].

### IV. NUMERIC EXAMPLE

In this study, the ground supported reinforced concrete rectangular tanks - endlessly long shipping channel is considered as shown in Fig. 3. The material characteristics of tank are: Young’s modulus \(E = 37\) GPa, Poisson ratio \(\nu = 0.20\), density \(\rho = 2550\) kg/m\(^3\). There is no roof slab structure covering the channel. The material characteristics of fluid filling (H\(_2\)O) are: bulk modulus \(B = 2.1\times10^9\) N/m\(^2\), density \(\rho_w = 1000\) kg/m\(^3\). As the excitation input we consider horizontal earthquake load given by the accelerogram of the earthquake in Loma Prieta, California (18.10.1989), Fig. 4. In the analysis we use just the accelerogram for the seismic excitation in \(y\) - direction.

Dynamic time-history response of concrete open top rectangular liquid storage tanks - shipping channel was performed by application of Finite Element Method (FEM) utilizing software ADINA. Arbitrary-Lagrangian-Eulerian (ALE) formulation was used for the problem. Two way Fluid-Structure Interaction (FSI) techniques were used for simulation of the interaction between the structure and the fluid at the common boundary. The solid walls and base of the shipping channel was modeled by using 3D SOLID finite element under plain strain condition. The fluid inside the shipping channel was modeled by using 3D FLUID finite elements. As the excitation input was considered the load of input time dependent horizontal displacement measured during the earthquake Loma Prieta in California, in Fig. 6.
FE-Model for 3D FSI analysis was shown in Fig. 5. FSI boundary is on the solid domain (black color) at the left side, and fluid domain on the right side, Fig. 5.

Fig. 6 input time dependent horizontal displacement measured during of earthquake Loma Prieta

Fig. 7 pressure of fluid in time 8 s

Fig. 8 pressure of fluid in time 21.36 s

The Fig. 7 presents distribution of pressure within fluid in time 8.0 s, whereas the same response of fluid shows Fig. 8 in time 21.36 s. In the time 8 s is influence hydrostatical pressure only and in time 21.36 s is time of obtained maximum pressure. The time dependent response of the pressure of fluid within time interval 17-27 s was described in Fig. 9 and Fig. 10, Fig. 9 in point "DL" (Down Left edge of fluid region) and Fig. 10 in point "DR" (Down Right edge of fluid region). It is seen that distribution of pressures in "DL" and "DR" points are asymmetric.

Fig. 9 time dependent response of the pressure of fluid in “DL” point

Fig. 10 time dependent response of the pressure of fluid in “DR” point

Fig. 11 shows more detail dependent responses of the fluid pressure within time interval 17-27 s together in points "DL", "DR" and "DM" (Down Middle of fluid region). In Fig. 12 is using smaller scale for dependent response of the fluid pressure within time interval 17-27 s in point "DM".

Fig. 11 time dependent response of the pressure of fluid in “DR”, “DL”, “DM” point

Fig. 12 time dependent response of the pressure of fluid in “DM” point

The Fig. 13 shows distribution of vertical displacement within fluid in time 21.56 s - the time of obtained maximum vertical displacement. The resulting time dependent vertical displacement of fluid in the point "UL" (Up Left edge of fluid region on free surface) was presented in Fig. 14, whereas the same response of fluid in point "UR" (Up Right edge of fluid region on free surface) was presented in Fig. 15.
region on free surface) was shown in Fig. 15. The timing of the peak response correlates well with peak excitation (Loma Prieta as in Fig. 4), which the numerical analysis makes realistic enough.

Fig. 13 shape of free surface and vertical displacement of fluid in time 21.56 s

Fig. 14 time dependent response of the vertical displacement of fluid in "UL" point

Fig. 15 time dependent response of the vertical displacement of fluid in “UR” point

Fig. 16 time dependent response of the vertical displacement of fluid in “UR”, “UL”, “UM” point

Fig. 17 time dependent response of the vertical displacement of fluid in "UM" point

Fig. 18 shape and Von Mises stress of tank in time t = 21.44 s

Fig. 19 time dependent response of tank relative horizontal displacement on the left corner

Fig. 20 time dependent response of tank relative horizontal displacement on the right corner

Fig. 16 shows dependent responses of the fluid vertical displacement within time interval 17-27 s together in points "UL", "UR" and "UM" (Up Middle of fluid region on free surface). It is seen that distribution of vertical displacement in "DL" and "DR" points are asymmetric. In Fig. 17 is using smaller scale for dependent response of the fluid vertical displacement within time interval 17-27 s in point "DM".
The Fig. 18 document distribution of Von Mises stress over the domain of interest in time $t = 21.44$ s, when peak responses were measured. The time dependent relative horizontal displacement of tank up corner to down corner was presented in Fig. 19 on the left wall, whereas the same response of fluid on the right wall in Fig. 20. Fig. 21 and Fig. 22 show dependent responses of relative horizontal of tank within time interval 17-27 s, Fig. 21 on left side and Fig. 22 on the right side.

![Fig. 21 time dependent response of tank relative horizontal displacement on the left corner within time interval 17-27 s](image1)

![Fig. 22 time dependent response of tank relative horizontal displacement on the right corner within time interval 17-27 s](image2)

V. CONCLUSION

The ground supported rectangular endlessly long top shipping channel was analyzed. The channel was excited by ground motion of Loma Prieta in California. Basic responses of the interest were: pressure in the fluid, displacement of the free fluid surface, structural deformation and stress distribution over the tank.

The peak hydrodynamic pressure and vertical displacement of fluid in the shipping channel along left side wall was similar, asymmetric and slightly higher to the peak values along the right wall.

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