An Inverse Calculation of Unimodular for Polynomial Matrices

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***Abstract*—The polynomial matrix approach is important for control system analysis and synthesis. In the approach, it sometimes needs the inverse calculation of unimodular. In this paper, we will propose an inverse calculation method of unimodular for polynomial matrices. Application to the solution of the generalized Bezout identity and zeros assignment of interactor will be also considered.**

I. INTRODUCTION

The control system analysis and synthesis by polynomial matrix approach are important especially in the area of parameter adaptive control systems [1], [2] and descriptor systems [3]. In the course of analysis and synthesis, it sometimes needs the inversion calculation of polynomial matrices. If the polynomial matrix is row (*or* column) proper [4], it can be easy to represent the inverse using the Structure Theorem [4]. If a given polynomial matrix is unimodular, then it is not easy to represent the inverse.

In this paper, we propose a calculation of inverse for unimodular matrices. The basic idea of the calculation is a derivation method for interactor matrices [5], [6]. So our propose method can be carried on by calculating Moore-Penrose pseudoinverse for some real matrices. Since there exists a program to calculate pseudoinverse in some standard software package for control systems, the method is simple and numerically stable.

The paper is organized as follows. In the next section, the problem statement will be given. In section 3, a calculation method will be presented using pseudoinverse. As applications, a solution of the generalized Bezout identity [7] and zeros assignment of interactor matrix will be considered in section 4. Concluding remarks will be given in section 5.

II. PROBLEM STATEMENT

A unimodular matrix $U(s)$ for polynomial matrices is defined as any square matrix which can be obtained from the identity matrix I by a finite number of elementary row and column operations on I, where the elementary row (column) operations on the polynomial matrix $D(s)$ with coefficients belongs in the set of real number $\Re$ are defined by

1) Interchange of rows (columns) i and j.
2) Multiplication of row (column) i by a nonzero scalar in $\Re$.
3) Replacement of row (column) i by itself plus any polynomial multiplied by any other row (column) j.

The determinant of a unimodular matrix is a nonzero scalar in $\Re$, and conversely, any polynomial whose determinant is a nonzero scalar in $\Re$ is a unimodular matrix. Therefore, the inverse of a unimodular matrix is also a unimodular matrix.

Define an $m \times m$ unimodular matrix $U(s)$ by

$$U(s) = U_0 + sU_1 + \cdots + s^w U_w \tag{1}$$

where w is a natural number and U_i ($i = 0,\ 1,\ \ldots, w$) are $m \times m$ real matrices. The problem considered in this paper is to propose a calculation method of the inverse of $U(s)$. If $U^{-1}(s)$ is given by

$$U^{-1}(s) = V_0 + sV_1 + \cdots + s^w V_w, \tag{2}$$

for real matrices V_i ($i = 0,\ 1,\ \ldots,\ w$), the problem becomes to find real matrices V_i. In the next section, it will be presented a calculation method of V_i.

III. AN INVERSE CALCULATION

From eqns.(1) and (2),

$$\begin{aligned} U(s)U^{-1}(s) &= I \\ &= \begin{bmatrix} I & sI & \cdots & s^{2w}I \end{bmatrix} \begin{bmatrix} U_0 & 0 & \cdots & 0 \\ U_1 & U_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ U_w & U_{w-1} & \cdots & U_0 \\ 0 & U_w & \cdots & U_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & U_w \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ \vdots \\ V_w \end{bmatrix}. \end{aligned} \tag{3}$$

Since the above equation holds for any s,

$$\begin{bmatrix} U_0 & 0 & \cdots & 0 \\ U_1 & U_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ U_w & U_{w-1} & \cdots & U_0 \\ 0 & U_w & \cdots & U_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & U_w \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ \vdots \\ V_w \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

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must hold. Especially, $U_0V_0 = I$ must hold and the solutions of the above equation are given by

$$\begin{aligned} V_0 &= U_0^{-1} \\ V_1 &= -U_0^{-1}U_1V_0 \\ V_2 &= -U_0^{-1}(U_2V_0 + U_1V_1) \\ &\vdots \\ V_w &= -U_0^{-1}(U_wV_0 + U_{w-1}V_1 + \cdots + U_1V_{w-1}) \end{aligned} \tag{5}$$

and the following relation must hold

$$\begin{bmatrix} U_w & U_{w-1} & \cdots & U_1 \\ 0 & U_w & \cdots & U_2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & U_w \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_w \end{bmatrix} = 0. \tag{6}$$

Conversely, the above condition can be used for the confirmation where a given polynomial matrix is unimodular or not.

Theorem 1 For a given polynomial matrix $U(s)$, which has coefficient matrices like eqn.(1), define real matrices V_0, V_1, …, V_w by eqn.(5). If V_i satisfy eqn.(6), then $U(s)$ is a unimodular matrix.

Example 1 Consider the following polynomial matrix $U(s)$:

$$U(s) = \begin{bmatrix} s+1 & s+2 \\ s+3 & s+4 \end{bmatrix}.$$

Define U_0 and U_1 by

$$U_0 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad U_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Then, V_0 and V_1 are given by

$$\begin{aligned} V_0 &= U_0^{-1} = \begin{bmatrix} 2 & -1 \\ -1.5 & 0.5 \end{bmatrix}, \\ V_1 &= -U_0^{-1}U_1V_0 = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix} \end{aligned}$$

and $U^{-1}(s)$ is given by

$$\begin{aligned} U^{-1}(s) &= V_0 + sV_1 \\ &= -\frac{1}{2}\begin{bmatrix} s+4 & -s-2 \\ -s-3 & s+1 \end{bmatrix} \end{aligned}$$

IV. Applications

A. The Generalized Bezout Identity

Let $(D(s),\ N(s))$ be right coprime polynomial matrices, with $D(s)$ nonsingular. Then, there exist polynomial matrices $\tilde{D}(s)$, $\tilde{N}(s)$, $X(s)$, $Y(s)$, $\tilde{X}(s)$ and $\tilde{Y}(s)$ such that

$$\begin{bmatrix} X(s) & Y(s) \\ \tilde{N}(s) & -\tilde{D}(s) \end{bmatrix} \begin{bmatrix} D(s) & \tilde{Y}(s) \\ N(s) & -\tilde{X}(s) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \tag{7}$$

Moreover, the block matrices in the above equation will all be unimodular. The above equation is called the generalized Bezout identity. In this section, we will consider a solution method of the above equation.

Assume that $D(s)$ is column proper and $N(s)D^{-1}(s)$ is strictly proper. Then, by the Structure Theorem [4], a controllability canonical realization of $N(s)D^{-1}(s)$, say $(A,\ B,\ C)$ can be obtained, i.e.,

$$sx(s) = Ax(s) + Bu(s), \tag{8}$$
$$y(s) = Cx(s) \tag{9}$$

where $u(s)$, $y(s)$ and $x(s)$ are input, output and state vector respectively. Let c_i denote the i-th row of C, and ν_1, …, ν_m be observability indices of $(C,\ A)$. Define $\nu := \max \nu_i$. Multiplying s eqn.(8) successively, employing eqn.(9) for substitution, we have

$$S_I^{\nu}(s)y(s) = \mathcal{O}_{\nu}(C,A)x(s) + \hat{\boldsymbol{T}}_{\nu-1}(C,A,B)S_I^{\nu-1}(s)u(s) \tag{10}$$

where

$$S_I^{\nu}(s) = \begin{bmatrix} I \\ sI \\ \vdots \\ s^{\nu}I \end{bmatrix}, \quad \mathcal{O}_{\nu}(C,A) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{\nu} \end{bmatrix},$$
$$\boldsymbol{T}_{\nu}(C,A,B) = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{\nu}B & CA^{\nu-1}B & \cdots & CB \end{bmatrix},$$
$$\hat{\boldsymbol{T}}_{\nu}(C,A,B) = \begin{bmatrix} 0_{m\times m(\nu+1)} \\ \boldsymbol{T}_{\nu}(C,A,B) \end{bmatrix}. \tag{11}$$

Since $(A,\ B,\ C)$ is a controllability canonical realization of $N(s)D^{-1}(s)$, there exists a partial state $\eta(s)$ such that

$$\begin{aligned} &x(s) = S^{\mu_i-1}(s)\eta(s), \\ &D(s)\eta(s) = u(s), \quad y(s) = N(s)\eta(s) \end{aligned} \tag{12}$$

where $S^{\mu_i-1}(s) = \text{block diag}\left\{\begin{matrix} 1 \\ s \\ \vdots \\ s^{\mu_i-1} \end{matrix}\right\}$. Substituting the above relations to eqn.(10) gives

$$\begin{aligned} &S_I^{\nu}(s)N(s)\eta(s) \\ &= \mathcal{O}_{\nu}(C,A)\eta(s) + \hat{\boldsymbol{T}}_{\nu-1}(C,A,B)S_I^{\nu-1}(s)D(s)\eta(s). \end{aligned} \tag{13}$$

Define

$$\hat{\mathcal{O}} = \begin{bmatrix} c_1 \\ \vdots \\ c_1A^{\nu_1-1} \\ c_2 \\ \vdots \\ c_mA^{\nu_m-1} \end{bmatrix}, \quad \tilde{\mathcal{O}} = \begin{bmatrix} c_1A^{\nu_1} \\ \cdots \\ c_mA^{\nu_m} \end{bmatrix}. \tag{14}$$

Then, from the definition of the observability indices, there exists a matrix Λ such that

$$\tilde{\mathcal{O}} = \Lambda\hat{\mathcal{O}}. \tag{15}$$

Using Λ, define

$$\tilde{\boldsymbol{D}} = [\ -\Lambda \quad I\ ]\,J, \tag{16}$$

where J is a row selection matrix such that

$$J\mathcal{O}_{\nu} = \begin{bmatrix} \hat{\mathcal{O}} \\ \tilde{\mathcal{O}} \end{bmatrix}. \tag{17}$$

Theorem 2 The polynomial matrices

$$\tilde{D}(s) = \tilde{\boldsymbol{D}} S_I^{\nu}(s), \quad \tilde{N}(s) = \tilde{\boldsymbol{D}} \hat{\boldsymbol{T}}_{\nu-1}(C, A, B) S_I^{\nu-1}(s) \tag{18}$$

are minimal degree solutions of

$$\tilde{D}(s) N(s) - \tilde{N}(s) D(s) = 0, \tag{19}$$

and $\tilde{D}(s)$ is a column proper polynomial matrix.

Now, choose linearly independent row vectors $c_1, \ldots, c_1 A^{\nu_1 - 1}, c_2, \ldots, c_m A^{\nu_m - 1}$, where

$c_1 A^{\nu_1}$ is row span of $c_1, \ldots, c_1 A^{\nu_1 - 1}$,

$c_2 A^{\nu_2}$ is row span of $c_1, \ldots, c_1 A^{\nu_1 - 1}, \; c_2, \ldots, c_2 A^{\nu_2 - 1}$,

$\vdots$

$c_m A^{\nu_m}$ is row span of $c_1, \ldots, c_m A^{\nu_m - 1}$.

Now we use the above procedure for polynomial matrices $D(s)$ and $\begin{bmatrix} N(s) \\ I \end{bmatrix}$. Let $(A, \; B, C_1)$ and $(A, \; B, C_2)$ denote a controllability canonical realization of $N(s) D^{-1}(s)$ and $D^{-1}(s)$ respectively. Since $D(s)$ and $N(s)$ are right coprime, C_2 is row span of $\hat{\mathcal{O}}$. Therefore, $\tilde{\mathcal{O}}$ is given by

$$\tilde{\mathcal{O}} = \begin{bmatrix} c_1 A^{\nu_1} \\ \vdots \\ c_m A^{\nu_m} \\ C_2 \end{bmatrix} \tag{20}$$

and thus $\tilde{\boldsymbol{D}} S_I^{\nu}(s)$ has the following structure:

$$\tilde{\boldsymbol{D}} S_I^{\nu}(s) = \begin{bmatrix} \tilde{D}(s) & 0 \\ -Y(s) & I \end{bmatrix}. \tag{21}$$

Then, a left annihilating polynomial matrix for $\begin{bmatrix} D^T(s) & N^T(s) & I \end{bmatrix}^T$ has the following structure:

$$\begin{bmatrix} \tilde{N}(s) & -\tilde{D}(s) & 0 \\ X(s) & Y(s) & -I \end{bmatrix}. \tag{22}$$

Example 2 Consider the following $D(s)$ and $N(s)$:

$$\begin{aligned} D(s) &= \begin{bmatrix} (s+1)(s+3) & 0 \\ 0 & (s+2)(s+4) \end{bmatrix}, \\ N(s) &= \begin{bmatrix} s+1 & s+2 \\ s+3 & s+4 \end{bmatrix}. \end{aligned}$$

Then, a controllability canonical realization of $\begin{bmatrix} N(s) \\ I \end{bmatrix} D^{-1}(s)$ is given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3 & -4 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -8 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 3 & 1 & 4 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

Then, by [9],

$$\begin{aligned} \begin{bmatrix} -\tilde{D}(s) & 0 \\ Y(s) & -I \end{bmatrix} &= \boldsymbol{D} \begin{bmatrix} I_4 \\ sI_4 \\ s^2 I_4 \end{bmatrix} \\ &= \begin{bmatrix} -s^2 - 7s - 12 & 0 & 0 & 0 \\ 0 & -s^2 - 3s - 2 & 0 & 0 \\ -0.5s - 2 & 0.5s + 1 & -1 & 0 \\ 0.5s + 1.5 & -0.5s - 0.5 & 0 & -1 \end{bmatrix} \end{aligned}$$

where

$$\boldsymbol{D} = \begin{bmatrix} 12 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ -2 & 1 & -1 & 0 & -0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.5 & -0.5 & 0 & -1 & 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

On the other hand, $\begin{bmatrix} \tilde{N}(s) \\ X(s) \end{bmatrix}$ is given by

$$\begin{aligned} \begin{bmatrix} \tilde{N}(s) \\ X(s) \end{bmatrix} &= \boldsymbol{D} \begin{bmatrix} 0 & 0 \\ \begin{bmatrix} C \\ I \end{bmatrix} B & 0 \\ \begin{bmatrix} C \\ I \end{bmatrix} AB & \begin{bmatrix} C \\ I \end{bmatrix} B \end{bmatrix} \begin{bmatrix} I \\ sI \end{bmatrix} \\ &= \begin{bmatrix} s+4 & s+3 \\ s+2 & s+1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

Therefore, the unimodular matrix $\begin{bmatrix} X(s) & Y(s) \\ \tilde{N}(s) & -\tilde{D}(s) \end{bmatrix}$ is given by

$$\begin{aligned} &\begin{bmatrix} X(s) & Y(s) \\ \tilde{N}(s) & -\tilde{D}(s) \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & -0.5s - 2 & 0.5 + 1 \\ 0 & 0 & 0.5s + 1.5 & -0.5s - 0.5 \\ s+4 & s+3 & -s^2 - 7s - 12 & 0 \\ s+2 & s+1 & 0 & -s^2 - 3s - 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & -2 & 1 \\ 0 & 0 & 1.5 & -0.5 \\ 4 & 3 & -12 & 0 \\ 2 & 1 & 0 & -2 \end{bmatrix} + s \begin{bmatrix} 0 & 0 & -0.5 & 0.5 \\ 0 & 0 & 0.5 & -0.5 \\ 1 & 1 & -7 & 0 \\ 1 & 1 & 0 & -3 \end{bmatrix} \\ &\quad + s^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \\ &=: \; U_0 + sU_1 + s^2 U_2. \end{aligned}$$

Define $\begin{bmatrix} X(s) & Y(s) \\ \tilde{N}(s) & -\tilde{D}(s) \end{bmatrix}^{-1} = V_0 + sV_1$ by

$$\begin{bmatrix} X(s) & Y(s) \\ \tilde{N}(s) & -\tilde{D}(s) \end{bmatrix}^{-1} = V_0 + sV_1 + s^2 V_2.$$

for some V_0, V_1 and V_2, then from the previous section,

$$\begin{aligned} V_0 &= U_0^{-1} = \begin{bmatrix} 3 & 0 & -0.5 & 1.5 \\ 0 & 8 & 1 & -2 \\ 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix}, \\ V_1 &= -U_0^{-1}U_1V_0 = -U_0^{-1}U_1U_0^{-1} \\ &= \begin{bmatrix} 4 & 0 & -0.5 & 0.5 \\ 0 & 6 & 0.5 & -0.5 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \\ V_2 &= -U_0^{-1}(U_1V_1 + U_2V_0) \\ &= U_0^{-1}(U_1U_0^{-1}U_1 - U_2)U_0^{-1} \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

For the above matrices, $U_2V_1 + U_1V_2 = U_2V_2 = 0$. Thus,

$$\begin{aligned} &\begin{bmatrix} X(s) & Y(s) \\ \tilde{N}(s) & -\tilde{D}(s) \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 3 & 0 & -0.5 & 1.5 \\ 0 & 8 & 1 & -2 \\ 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix} + s \begin{bmatrix} 4 & 0 & -0.5 & 0.5 \\ 0 & 6 & 0.5 & -0.5 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \\ &\quad + s^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} s^2+4s+3 & 0 & -0.5s-0.5 & 0.5s+1.5 \\ 0 & s^2+6s+8 & 0.5s+1 & -0.5s-2 \\ s+1 & s+2 & 0 & 0 \\ s+3 & s+4 & 0 & 0 \end{bmatrix}. \end{aligned}$$

B. Zeros Assignment of Interactor Matrix

For a given $m \times m$ nonsingular transfer function matrix $G(s)$, there exists a nonsingular polynomial matrix $L(s)$ such that

$$\lim_{s\to\infty} L(s)G(s) = I. \tag{23}$$

$L(s)$ is called an interactor matrix for $G(s)$ [5]. The authors presented a simple derivation method of an interactor, which zeros lie at the origin. In this subsection, we will propose a method to assign the zeros of interactor arbitrary.

Let $(A,\ B,\ C)$ denote a realization of a given $G(s)$ Assume that $L(s)$ has the following structure:

$$L(s) = sL_1 + s^2L_2 + \cdots + s^wL_w := s\boldsymbol{L}S_I^{w-1}(s), \tag{24}$$

where w is the positive integer. Then, the coefficient matrix $\boldsymbol{L}$ is given by

$$\boldsymbol{L} = \begin{bmatrix} I & 0_{m\times m(w-1)} \end{bmatrix} \boldsymbol{T}_{\boldsymbol{w-1}}^{\dagger}(C, A, B) \tag{25}$$

(detail discussion can be found in [6]).

The feedback gain matrix F which makes the closed-loop transfer function matrix $G_{cl}(s) = L^{-1}(s)$ (after possibly poles-zeros cancellation) is given by

$$F = \boldsymbol{L}\mathcal{O}_{w-1}(C, A)A \tag{26}$$

[10]. Define

$$A_F := A - BF. \tag{27}$$

Then, $G_{cl}(s) = C(sI - A_F)^{-1}B$. Let nu denote the observability index for $(C,\ A_F)$. Calculate a minimal left annihilating matrix $\tilde{D}$ for $\mathcal{O}_\nu(C, A_F)$ as in the previous subsection. Then,

$$\tilde{D}(s) := \tilde{\boldsymbol{D}}S_I^{\nu}(s) \tag{28}$$

is column proper and

$$\tilde{N}(s) := \tilde{\boldsymbol{D}}\hat{\boldsymbol{T}}_{\nu-1}(C, A_F, B)S_I^{\nu-1}(s) \tag{29}$$

is a unimodular matrix, and $L(s)$ can be written by

$$L(s) = \tilde{N}^{-1}(s)\tilde{D}(s). \tag{30}$$

Calculate a right coprime factorization of $\tilde{D}^{-1}(s)\tilde{N}(s)$, say, $N(s)D^{-1}(s)$ where $D(s)$ is column proper. So the above equation can be rewritten by

$$L(s) = D(s)N^{-1}(s). \tag{31}$$

Note that $N(s)$ is a unimodular matrix. Set a desired polynomial matrix $D_*(s)$ which has the same column degree with $D(s)$, i.e.,

$$\lim_{s\to\infty} D(s)D_*^{-1}(s) = K\ (nonsingular). \tag{32}$$

Define

$$L_*(s) = D_*(s)N^{-1}(s). \tag{33}$$

Then,

$$\begin{aligned} \lim_{s\to\infty} L_*(s)G(s) &= \lim_{s\to\infty} D_*(s)\cdot D^{-1}(s)D(s)\cdot N^{-1}(s)G(s) \\ &= \lim_{s\to\infty} D_*(s)D^{-1}(s)\cdot L(s)G(s) \\ &= K\ (nonsingular) \end{aligned} \tag{34}$$

i.e., $L_*(s)$ is an interactor matrix with desired zeros. Therefore, the inverse calculation given in section III for $N(s)$ is useful.

V. Conclusions

In this paper, we consider an inverse calculation of unimodular matrix for polynomial matrices. Since the method is based on the pseudoinverse calculation, it is numerically stable. Application to the solution of the generalized Bezout identity and the zeros assignment of interactor matrix were also considered.

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