A Simple Dual Method for Optimal Allocation of Total Network Resources

I.V. Konnov, A.Yu. Kashuba, E. Laitinen

***Abstract*—We consider a general problem of optimal allocation of a homogeneous resource (bandwidth) in a wireless communication network, which is decomposed into several zones (clusters). Due to instability of users requirements, the fixed network resource volume may be not sufficient in some time periods, hence the network manager can buy additional volumes of this resource. This approach leads to a constrained convex optimization problem. We suggest the dual Lagrangian method to be applied to a selected constraints. This enables us to replace the initial problem with one-dimensional dual one. We consider the case of the affine cost (utility) functions, when each calculation of the value of the dual function requires solution of a special linear programming problem. The results of the numerical experiments confirm the preferences of the new method over the previous ones.**

***Index Terms*—Resource allocation, wireless networks, bandwidth, zonal network, dual Lagrange method, linear programming.**

I. Introduction

THe current development of telecommunication systems creates a number of new challenges of efficient management mechanisms involving various aspects. One of them is the efficient allocation of limited communication networks resources. In fact, despite the existence of powerful processing and transmission devices, increasing demand of different communication services and its variability in time, place, and quality, leads to serious congestion effects and inefficient utilization of significant network resources (e.g., bandwidth and batteries capacity), especially in wireless telecommunication networks. This situation forces one to replace the fixed allocation rules with more flexible mechanisms; see e.g. [1]–[4]. Naturally, treatment of these very complicated systems is often based on a proper decomposition/clustering approach, which can involve zonal, time, frequency and other decomposition procedures for nodes/units; see e.g. [5], [6]. In [7], [8], several optimal resource allocation problems in telecommunication networks and proper decomposition based methods were suggested. A further development of these models, where a system manager can utilize additional external resources for satisfying current users resource requirements, was considered in [9]. We note that such a strategy is rather typical for contemporary wireless communication networks, where WiFi or femtocell communication services are utilized in addition to the usual network resources; see e.g. [10]. A decomposition method for solution of the arising optimization problem was also suggested in [9]. It was based on an explicit volume resource allocation procedure and gave a multi-level iterative procedure. In this paper, we consider some other approach to enhance the performance of the solution method. It consists in utilization of the Lagrangian multipliers only for the total resource bound, which yields an one-dimensional dual optimization problem. We consider the case of the affine cost (utility) functions, when each calculation of the value of the dual function requires solution of a special linear programming problem. The results of the numerical experiments confirms the preferences of the new method over the previous ones.

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II. Problem description

Let us consider a network with nodes (attributed to users), which is divided into n zones (clusters) within some fixed time period. For the k-th zone ($k = 1, \dots, n$), I_k denotes the index set of nodes (currently) located in this zone, b_k is the maximal fixed resource value. The network manager satisfies users resource requirements in the k-th zone by allocation of the own (inner) resource value $x_k \in [0, b_k]$ and also by taking the external resource value $z_k \in [0, c_k]$. Clearly, these values require proper maintenance expenses $f_k(x_k)$ and side payments $h_k(z_k)$ for each $k = 1, \dots, n$. We suppose also that there exists the upper bound B for the total amount of the inner resource of the network. Next, if the i-th user receives the resource amount y_i with the upper bound a_i, then he/she pays the charge $\varphi_i(y_i)$. The problem of the network manager is to find an optimal allocation of the resource among the zones and can be written as follows:

$$\max_{(x,y,z)\in W, \sum\limits_{k=1}^{n} x_k \leq B} \to \mu(x, y, z) \tag{1}$$

where

$$\mu(x, y, z) = \sum_{k=1}^{n} \left[\sum_{i\in I_k} \varphi_i(y_i) - f_k(x_k) - h_k(z_k) \right], \tag{2}$$

and

$$W = \left\{ (x, y, z) \;\middle|\; \begin{array}{c} \sum_{i\in I_k} y_i = x_k + z_k,\ 0 \leq y_i \leq a_i,\ i \in I_k, \\ 0 \leq x_k \leq b_k,\ 0 \leq z_k \leq c_k,\ k = 1, \dots, n \end{array} \right\}. \tag{3}$$

In what follows we assume all the functions $\varphi_i(y_i)$, $f_k(x_k)$, and $h_k(z_k)$ are affine, i.e.

$$\begin{array}{rcl} \varphi_i(y_i) & = & \alpha'_i y_i + \alpha''_i,\ \alpha'_i > 0,\ i \in I_k,\ k = 1, \dots, n, \\ f_k(x_k) & = & \beta'_k x_k + \beta''_k,\ \beta'_k > 0,\ k = 1, \dots, n, \\ h_k(z_k) & = & \gamma'_k z_k + \gamma''_k,\ \gamma'_k > 0,\ k = 1, \dots, n. \end{array}$$

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III. Solution method

Let us define the Lagrange function of problem (1)–(3) as follows:

$$L(x,u,z,\lambda)=\mu(x,y,z)-\lambda\left(\sum_{k=1}^{n}x_k-B\right).$$

We utilize the Lagrangian multiplier λ only for the total resource bound. We can now replace problem (1)–(3) with its one-dimensional dual:

$$\min_{\lambda\geq 0}\rightarrow\psi(\lambda), \tag{4}$$

where

$$\begin{aligned}\psi(\lambda) &= \max_{(x,y,z)\,\in\, W} L(x,y,z,\lambda)=\lambda B\\ &+ \max_{(x,y,z)\,\in\, W}\sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i)-(f_k(x_k)+\lambda x_k)-h_k(z_k)\right]\end{aligned}$$

Its solution can be found by one of well-known single-dimensional optimization problem.

In order to calculate the value of $\psi(\lambda)$ we have to solve the inner problem:

$$\max\rightarrow\sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i)-(f_k(x_k)+\lambda x_k)-h_k(z_k)\right]$$

subject to

$$\sum_{i\in I_k}y_i=x_k+z_k,\ 0\leq y_i\leq a_i,\ i\in I_k,$$
$$0\leq x_k\leq b_k,\ 0\leq z_k\leq c_k,\ k=1,\dots,n.$$

Obviously, this problem decomposes into n independent zonal linear programming problems

$$\max\rightarrow\left[\sum_{i\in I_k}\varphi_i(y_i)-(f_k(x_k)+\lambda x_k)-h_k(z_k)\right], \tag{5}$$

$$\sum_{i\in I_k}y_i=x_k+z_k,\ 0\leq y_i\leq a_i,\ i\in I_k, \tag{6}$$

$$0\leq x_k\leq b_k,\ 0\leq z_k\leq c_k, \tag{7}$$

for $k=1,\dots,n$. Note that the cost function in (5) is rewritten as

$$\sum_{i\in I_k}\alpha_i' y_i-(\beta_k'+\lambda)x_k-\gamma_k' z_k.$$

It follows that we can find very easily an exact solution of each of problems (5)–(7) in a finite number of iterations by a simple ordering algorithm.

This approach gives an alternative to the method from [9], which involved explicit marginal profit values for each zone depending on its resource allocation share. Then one can also replace the initial problem (1)–(3) with a sequence of one-dimensional ones, but that requires a multi-level iterative procedure with concordance of accuracies at each level. Therefore, our new approach simplifies essentially that from [9].

TABLE I
Results of testing with $J=510$, $n=70$, $\delta=10^{-2}$

ε	N_ε	T_ε (DML)	T_ε (SDM)
10^{-1}	20	3.3907	0.0050
10^{-2}	24	3.9427	0.0038
10^{-3}	29	4.9633	0.0043
10^{-4}	34	5.7347	0.0057

TABLE II
Results of testing with $n=70$, $\varepsilon=10^{-2}$, $\delta=10^{-2}$

J	N_ε	T_ε (DML)	T_ε (DMLA)	T_ε (SDM)
210	24	1.7453	1.2240	0.0009
310	24	2.4480	1.7967	0.0025
410	24	3.1980	2.3910	0.0028
510	24	3.9427	2.9007	0.0038
610	24	4.6097	3.4167	0.0038
710	24	5.3070	3.9220	0.0040
810	24	6.0260	4.4427	0.0031
910	24	6.9170	4.9533	0.0047
1010	24	7.4843	5.4797	0.0047

IV. Numerical experiments

In order to evaluate the performance of the new method denoted as (SDM) and to compare it with that from [9] denoted as (DML) we made a number of computational experiments.

We utilized the golden section method for solving the single-dimensional optimization problems. The methods were implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial intervals for choosing the dual variable λ (and the additional dual variables in (DML)) were taken as [0,1000]. The initial intervals for choosing the zonal allocation shares u_k in (DML) were taken as $[0,R]$ with $R=B+\sum_{k=1}^{n}c_k$, B was chosen to be 1000. Values of b_k and c_k were chosen by trigonometric functions in $[1,11]$, values of a_i were chosen by trigonometric functions in $[1,2]$. The number of zones was varied from 5 to 105, the number of users was varied from 210 to 1010. Users were distributed in zones either uniformly or according to the normal distribution. The processor time and number of iterations, which were necessary to find an approximate solution of problem (4) within the same accuracy, were not significantly different for these two cases of distributions.

Further we report the results of tests, which include the time and number of iterations needed to find a solution of problem (4) within some accuracies. Let ε and δ denote the desired accuracy of finding a solution of problem (4) and solutions of auxiliary inner problems in (DML). Let J denote the total number of users, N_ε the number of upper iterations in λ, T_ε the total processor time in seconds. For the same accuracy, both the methods gave the same numbers of upper iterations, so that the main difference was in the processor time. The results of computations are given in Tables I–III. We inserted also the results for (DML) with adaptive strategy of choosing the inner accuracies. We named by (DMLA) this version of the method. In Table I, we vary the accuracy ε, in Tables II and III we vary the total number of users and the number of zones, respectively. From the results we can conclude that the

TABLE III
RESULTS OF TESTING WITH $J = 510$, $\varepsilon = 10^{-2}$, $\delta = 10^{-2}$

n	N_ε	T_ε (DML)	T_ε (DMLA)	T_ε (SDM)
5	24	3.5730	2.6517	0.0032
15	24	3.6200	2.6877	0.0019
25	24	3.6927	2.7240	0.0016
35	24	3.7500	2.7917	0.0013
45	24	3.7970	2.7970	0.0034
55	24	3.8487	2.8383	0.0034
65	24	3.9480	2.8857	0.0044
75	24	3.9740	2.9167	0.0047
85	24	4.0210	2.9530	0.0038
95	24	4.1720	3.0260	0.0035
105	24	4.2187	3.0467	0.0053

new method (SDM) has the significant preference over that in [9], which enables us to apply (SDM) for online solution of these resource allocation problems.

V. CONCLUSION

In this work, we considered a problem of managing limited resources in a zonal wireless communication network and gave its constrained convex optimization problem formulation. We proposed a new dual Lagrangian method, which was applied to the case of the affine cost (utility) functions. The results of the numerical experiments confirmed the preferences of this method over the previous ones.

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